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SHEAR BEAM MODEL FOR INTERFACE FAILURE UNDER ANTIPLANE SHEAR (I) —FUNDAMENTAL BEHAVIOR*

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Abstract: The propagation of interlayer cracks and the resulting failure of the interface is a typical mode occurring in rock engineering and masonry structure. On the basis of the theory of elastoplasticity and fracture mechanics, the shear beam model for the solution of interface failure was presented. The concept of 'cohesive crack' was adopted to describe the constitutive behavior of the cohesive interfacial layer. Related fundamental equations such as equilibrium equation, constitutive equations were presented. The behavior of a double shear beam bonded through cohesive layer was analytically calculated. The stable propagation of interface crack and process zone was investigated.

Key words: interface layer; cohesive layer; anti-plane shear; shear beam model; failure; instability; damage

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Introduction

The propagation of interlayer cracks and the resulting failure of the interface is one of more important modes occurring in composite materials, rocks and ceramics. A detailed survey of research in this area can be found, for instance, in the articles by Garg[1] or Hutchinson and Suo[2] who discussed mixed-mode crack propagation using the Griffith energy condition. In this paper, we shall apply the cohesive crack[3,4] model assuming the existence of a damage process zone ahead of the crack. The interface layer in this zone is assumed to undergo an elasto plastic deformation and damage inducing plastic softening. When the compressive normal traction acts at the interface, the frictional slip occurs along the cracked portion. For cyclic loading a set of progressive and reverse slip zones develop at the interface and frictional hysteretic effects occur accompanied by contact dissipation and evolution of the state of delaminated interface due to wear. The coupling between the propagation of damage zone and the frictional dissipation may

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then occur.

However, the effects of interface friction have not been fully investigated. In general, several major topics should be investigated, namely: ① formulation of slip and wear rules at the delaminated interface portions; ② damage zone evolution rules and also localized temperature effects due to cyclic slip and interface dissipation; ③ the propagation and instability of interface crack and its influence to entire structure.

The present work is aimed at the analytical solution of the anti-plane shear damage growth at the interface layer between the plate and rigid foundation, assuming the compressive normal traction acting on the interface. The problem is simplified by neglecting the minor shear stress effect in the transverse direction and the concept of shear beam model is presented for the solution of interface failure. Analytical results are obtained for the distribution of shear stress and displacement fields corresponding to the stage of stable propagation of crack. The instability character caused by softening interfacial material of the laminated structure and interaction between the free end of the beam and process zone on some equilibrium paths can be found in the second part of the present work.

1 Problem Formulation

In this section, the anti-plane shear problem in the theory of elasticity is introduced first, and then the concept of shear beam model is presented.

Consider a double-shear-plate of length L, width b, and thickness t, bonded by cohesive layer of thickness $2h$, $h \ll t$, as shown in Fig. 1.

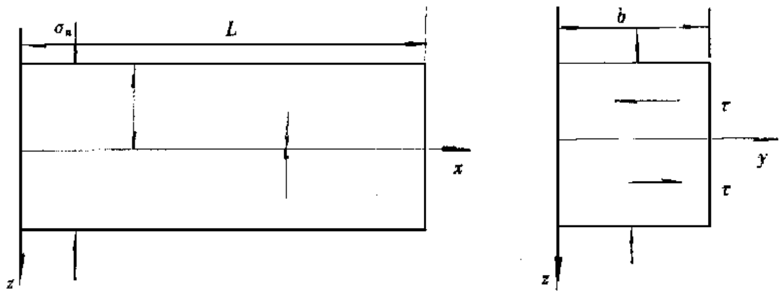

Fig. 1 Double-shear-plates bonded by cohesive layer

The uniform compressive traction $\sigma_{zz} = \sigma_n$ is assumed to act at the upper plate surface. The plate is loaded by the shear force $\tau(0) = \tau_{yx}(0)A$, $A = bt$, at the end section $x = 0$. The other end section at $x = L$ remains traction free. Assuming the transverse dimension b to be much larger than the plate thickness t, the flexural effect can be neglected and the state of anti-plane shear can be assumed with two non-vanishing shear stress component τ_{yx} and τ_{yz}, so that the equilibrium equation is

$$\partial \tau_{yx}/\partial x + \partial \tau_{yz}/\partial z = 0, \tag{1}$$

and $\sigma_{zz} = \sigma_n$ =const is the initial stress induced by the lateral compressive traction along the z-axis. Denoting by $w = w(x, z)$, the displacement field along the y-axis and using the Hooke's law:

$$\tau_{yz} = -G_2(\partial w/\partial z), \quad \tau_{xy} = -G_1(\partial w/\partial x). \tag{2}$$

The equilibrium equation (1) takes the form

$$G_1(\partial^2 w/\partial x^2) + G_2(\partial^2 w/\partial z^2) = 0, \tag{3}$$

where G_1, G_2 are the shear moduli along x- and z- axes. The boundary conditions at the interface $z=0$, $\tau_{yz} = -\tau_f$, upper boundary $z=t$, $\tau_{yz}=0$, and at the transverse boundaries $x=0$: $\tau_{xy}(0) = f_0(y,z)$, $x=L$: $\tau_{xy}(L)=0$ should be satisfied.

A simplified solution can be generated by assuming the distribution of τ_{yz}, namely

$$\tau_{yz} = \tau_f(x)\left((z/t) - 1\right), \tag{4}$$

where $\tau_f(x)$ is the interface shear stress at $z = 0$. Denoting $\tau_{xy} = \tau$, the equilibrium Eq. (1) takes the form

$$\partial\tau/\partial x + \tau_f/t = 0. \tag{5}$$

Eq. (5) is the equilibrium equation for anti-plane shear problem simplified by Eq. (4).

Let us note that this form of Eq. (5) can be obtained by assuming the shear beam model, as shown in Fig. 2, that is assuming $w = w(x)$, i.e., $\partial w/\partial x = 0$ and the beam is an elastic beam,

$$\tau = -G\mathrm{d}w/\mathrm{d}x. \tag{6}$$

The shear stress τ of the elastic beam and shear stress τ_f of cohesive layer form a self-balanced system as that described in Reference [5]. Writing the equilibrium equation for the elastic beam interacting with the interface cohesive layer

$$A\mathrm{d}\tau/\mathrm{d}x + b\tau_f = 0, \tag{7}$$

where $A = bt$ denotes the transverse cross-section area. This equation can be rewritten as follows

$$\mathrm{d}^2 w/\mathrm{d}x^2 - b\tau_f/AG = \mathrm{d}^2 w/\mathrm{d}x^2 - \tau_f/tG = 0. \tag{8}$$

Comparing Eq. (7) with Eq. (5), it can be concluded that the shear beam model is equivalent to the interfacial anti-plane shear problems under uniform lateral compression.

To complete the problem formulation, we shall present the constitutive equations for the interface layer of thickness h. Denoting the shear strain in the layer by γ_f, we have for the elastic interface layer of thickness h. Denoting the shear strain in the layer by γ_f, we have for the elastic response

$$\gamma_f = w/h, \quad \tau_f = G_f\gamma_f = G_f w/h = Kw, \tag{9}$$

where G_f denotes the shear modulus of the interface layer.

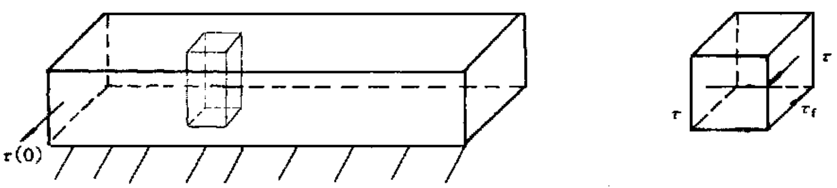

Fig. 2 Shear beam model

For the fully damaged layer, the frictional slip occurs when the shear stress reaches the limit value $\tau_s = \mu\sigma_n$, where μ is the coefficient of friction. The total displacement and its rate are decomposed into elastic and slip components, thus

$$w = w^e + w^s, \quad \dot{w} = \dot{w}^e + \dot{w}^s, \tag{10}$$

and the limit friction condition is

$$F^s(\sigma_n, \tau_f) = \| \tau_f \| - \mu\sigma_n \leq 0. \tag{11}$$

For specified σ_n, the shear stress first reaches the maximum critical value τ_{fc} and next in the elastoplastic softening process, decreases to the limit friction value $\tau_s = \sigma_n$ in the fully damaged state. We can therefore write

$$F^s(\sigma_n, \tau_f) = \| \tau_f \| - \mu\sigma_n - \tau_{fc}^0 \leq 0, \tag{12}$$

where τ_{fc}^0 is the critical stress value for $\sigma_n = 0$. In the tension zone, the damage surface, as shown in Fig. 3(a), can be assumed in the form

$$F_t^d = \left[\left(\sigma_n / \sigma_{t0} \right) + \left(\tau_n / \tau_{fc}^0 \right) \right]^{\frac{1}{2}} - 1 \leq 0. \tag{13}$$

As the contact dilatancy is neglected, the plastic potential is identical to

$$\Phi = \tau_f - \alpha = 0, \tag{14}$$

where α is material constant. The discussion of a dilatant contact condition can be found in the paper by Mroz and Seweryn[6]. So the slip rule is

$$\dot{w}^s = \lambda^s \frac{\partial \Phi}{\partial \tau_f} = \lambda^p > 0, \quad F^s \leq 0, \quad \lambda^p F^s = 0, \tag{15}$$

where λ^p is the positive slip multiplier. For monotonic loading, Eq. (15) can be integrated to provide the total slip displacement

$$w^s = \lambda^p \frac{\partial \Phi}{\partial \tau_f} = \lambda^p > 0, \quad F^s \leq 0, \quad \lambda^p F^s = 0. \tag{16}$$

However, when unloading and reverse slip occurs, the memory of the previous slip displacement must be stored and added to the reverse slip displacement. Fig. 3 presents the initial and limit friction surface as in the plane of τ_f-σ_n, and the stress-displacement diagram.

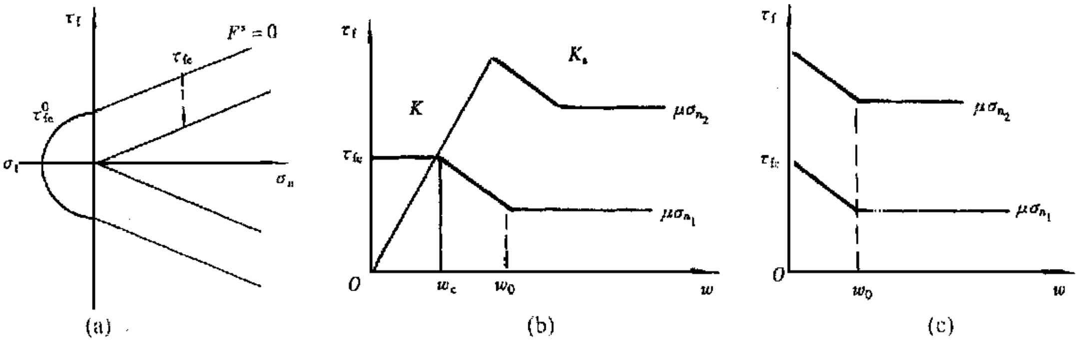

Fig. 3 (a) Critical and limit friction condition,
(b) Stress-strain relation for elastic cohesive layer,
(c) Stress-strain relation for rigid cohesive layer

For specified σ_n, the shear stress first reaches the maximum critical value τ_{fc} and next in the elastoplastic softening process, decreases to the limit friction value $\tau_s = \mu\sigma_n$ in the fully damaged state. We can therefore write

$$\left.\begin{aligned} &\tau_f = Kw, \; 0 < w < w_c = \frac{\tau_{fc}}{K}, \quad \tau_f = \tau_{fc} - K_s(w - w_c), \\ &w_c < w < w_0 = w_c + \frac{\tau_{fc} - \tau_s}{K_s}, \quad \tau_f = \tau_s = \mu\sigma_n, w_0 < w, \end{aligned}\right\} \tag{17}$$

where K_s is the elastoplastic softening modulus. The total dissipated work can be decomposed into damage and friction dissipation, thus

$$W = W^d + W^p, \tag{18}$$

where

$$W^p = w^s \tau_s, \; W^d = \int_{w_c} (\tau_f - \mu\sigma_n) \, dw, \tag{19}$$

as is shown in Fig. 4, W^p is the area of parallelogram OEFCO; W^d is the area of quadrilateral AHGBA.

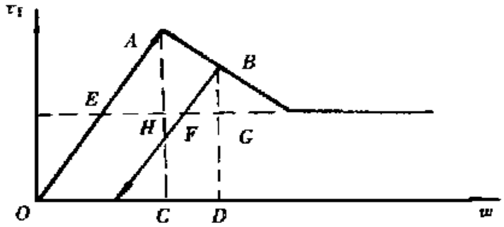

Fig. 4 Schematic representation of energy dissipation

2 Solution for a Rigid-Damaging Interface

2.1 Rigid-damaging interface model

Rigid-damaging interface model is the case $K = G_f / h = \infty$, i.e., neglecting the elasticity of the interface layer. The stress-strain response of the interface for this case is presented in Fig. 3(c).

Introduce the non-dimensional length variables

$$x = \bar{x}/t, \; w = \bar{w}/t, \; h = \bar{h}/t, \; s = \bar{s}/t, \; L = \bar{L}/t \tag{20}$$

referred to the plate thickness t. The equilibrium equation Eq. (7) and Eq. (8) take the form

$$d\tau/dx + \tau_f = 0, \quad d^2 w/dx^2 - \tau_f/G = 0. \tag{21}$$

Assume the monotonically growing shear stress $\tau(0)$ to be applied at the plate end $x = 0$, the associated end displacement is denoted by $w(0)$. Along the length of the shear beam, in the interface layer, the damage zone starts to propagate from this end followed by the slip zone for increasing $w(0)$. The following consecutive stages can be distinguished:

1) Damage initiation stage with the damage zone developing from the loaded end $x = 0$.
2) Crack propagation stage with both slip and damage zone propagation along the plate.
3) End-zone stage with damage zone interacting with the end boundary $x = L$.

In the following section, the stage 2 will be discussed only and the study on the topics of stage 1 and 2 mentioned above can be found in the second part of this work.

2.2 Slip and damage zone propagation

Let us first discuss the case of developed frictional slip and damage zones at the interface. The position of crack tip, i.e., the frontier of frictional slipping zone, is represented by S_1, and the position of the frontier of softening damage zone is represented by S_2. Along the whole length of interfacial cohesive layer, it can be divided into three regions, i.e., the frictional sliding zone, damage-softening zone, and rigid zone. In the rigid zone, both stress and displacement are

0. In the following, solution will be presented for the frictional sliding zone and damage-softening zone.

2.2.1 Solution for the frictional sliding zone

Assuming continuity of the interface shear stress in the slip and damage zones, so that

$$[w] = [w'] = [w''] = 0, \text{ for } 0 \leq x \leq S_1, \tag{22}$$

where square bracket denotes the discontinuity and w' w'' are the first- and second-order derivatives of w with respect to x. The condition $w'' = 0$ does not occur at $x = S_2$ as τ_f changes discontinuously, $\tau_f(S_2^-) = \tau_{fc}$, $\tau_f(S_2^+) = 0$. Within the slipping zone $0 \leq x \leq S_1$:

$$\tau_f = \tau_s = \mu\sigma_n, \tag{23}$$

and Eq. (21) provides the stress and displacement fields

$$\left.\begin{aligned} w(x) &= -\frac{\mu\sigma_n}{G}\left(\frac{x^2}{2} - S_1 x + \frac{S_1^2}{2}\right) + \frac{\tau(S_1)}{G}(S_1 - x) + w(S_1), \\ \tau(x) &= \mu\sigma_n(x - S_1) + \tau(S_1), \end{aligned}\right\} \tag{24}$$

where $\tau(S_1)$ and $w(S_1)$ are the shear stress and displacement at point $x = S_1$. Actually $w(S_1)$ is the critical displacement value of COD criterion. From the damage-softening constitutive equation, the following relationship is obtained

$$w(S_1) = \frac{\tau_{fc} - \mu\sigma_n}{K_s} = \frac{\tau_{fc}^0}{K_s} = w_c, \quad \tau(S_1) = \tau(0) - \mu\sigma_n S_1. \tag{25}$$

2.2.2 Solution for the damage-softening zone

Within the damage zone $S_1 \leq x \leq S_2$, the constitutive relationship is

$$\tau_f = \tau_{fc} - K_s(w - w_c), \tag{26}$$

and the equilibrium equation takes the form

$$\frac{d^2 w}{d x^2} + \frac{K_s}{G}w = \frac{\tau_f}{tG}. \tag{27}$$

Now the stress and displacement fields are expressed as follows:

$$\left.\begin{aligned} w(x) &= c_1\cos(r_s x) + c_2\sin(r_s x) + \frac{\tau_{fc}}{K_s}, \\ \tau(x) &= Gr_s[(c_1\sin(r_s x) - c_2\cos(r_s x)], \end{aligned}\right\} \tag{28}$$

where

$$r_s = \sqrt{\frac{K_s}{G}}, \tag{29}$$

and c_1 and c_2 are the integration constants that can be specified from the boundary conditions

$$\tau(S_2) = 0, \quad w(S_2) = 0, \tag{30}$$

so we obtain

$$c_1 = \frac{\tau_{fc}}{K_s}\cos(r_s S_2), \quad c_2 = \frac{\tau_{fc}}{K_s}\sin(r_s S_2). \tag{31}$$

Substituting Eq. (31) into Eq. (28), we obtain

$$\left.\begin{aligned} w(x) &= \frac{\tau_{fc}}{K_s}\{1 - \cos[r_s(S_2 - x)]\}, \\ \tau(x) &= Gr_s\sin[r_s(S_2 - x)]. \end{aligned}\right\} \tag{32}$$

Substituting the first equation of Eq. (32) into Eq. (26), we can obtain the equation of $\tau_f(x)$,

Fig. 5(a), (b), (c) show the distribution of $w(x)$, $\tau(x)$, $\tau_f(x)$ along the whole length during the stable propagation of crack. Fig. 5 indicates that the results obtained by the theory of shear beam model is not like the results obtained by fracture mechanics which is of high singularity at crack tip, but is of higher stress concentration compared with the results obtained by the theory of plasticity. The results shown in Fig. 5 are in good accordance with engineering experience.

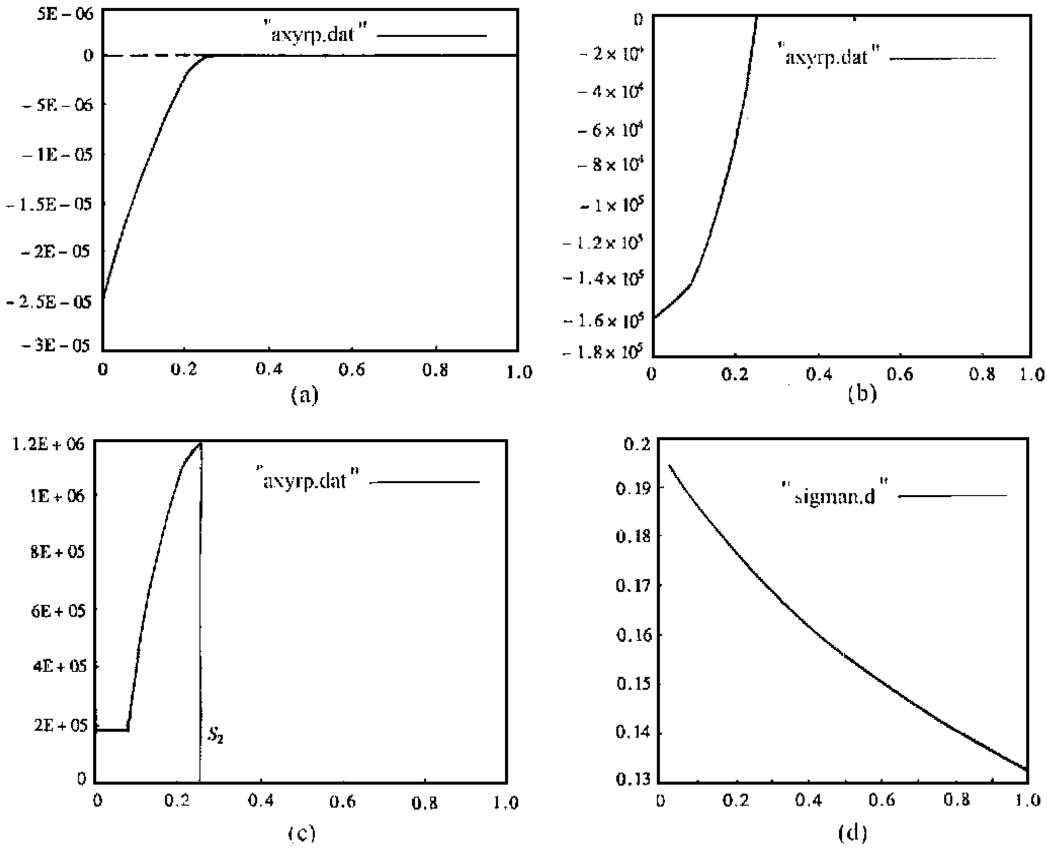

Fig. 5 Along the length of the beam, distribution of (a) $w(x)$; (b) $\tau(x)$; (c) $\tau_f(x)$; (d) The variation of S_d with η

2.2.3 The length of damage zone

The length of damage zone is one of the principal parameters in this study. Satisfying the continuity conditions expressed in Eq. (22) at $x = S_1$, we obtain

$$S_d = S_2 - S_1 = \frac{1}{r_s}\cos^{-1}\left(\frac{\mu\sigma_n}{\tau_{fc}}\right). \tag{33}$$

The length of damage zone $S_d = S_2 - S_2$ is constant during stable propagation stage and is expressed by Eq. (33). The value of S_d decreases for increasing ratio $\eta = \mu\sigma_n / \tau_{fc}^0$ and is inversely proportional to r_s. Fig. 5(d) illustrates the dependence of S_d on two parameter η (given $r_s = 10$). The value of S_1 is related to $\tau(0)$ and $w(0)$ by the equation

$$\left.\begin{aligned} \frac{\tau(0)}{G} &= \tau_s \frac{\tau_{fc}}{K_s}\sin(r_s S_d) + \frac{\mu\sigma_n}{G}S_1, \\ w(0) &= -\frac{\mu\sigma_n}{2G}S_1^2 + \frac{\tau}{G}S_1 + w(S_1), \end{aligned}\right\} \tag{34}$$

and we obtain

$$w(S_1) = \frac{\tau_{fc} - \mu\sigma_n}{K_s} = \frac{\tau_{fc}^0}{K_s} = w_c, \qquad \tau(S_1) = \tau(0) - \mu\sigma_n S_1. \tag{35}$$

3 Conclusion

The shear beam model is presented in this paper for the fracture failure of interface. The relevant fundamental equations such as constitutive equation, equilibrium equation, and incremental relationships are presented. Calculation is carried out for the plates bonded through cohesive layer under the actions of anti-plane shear and lateral compression. Analytical solutions are obtained for the stress and displacement during the stable propagation of interfacial crack. Results indicate that results obtained by the theory of shear beam model presented above are not like the results obtained by fracture mechanics which is of high singularity at crack tip, but is of much higher stress concentration than that obtained by the theory of plasticity.

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