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Damage Model for Jointed Rock Mass and Its Application to Tunnelling

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ABSTRACT

This paper establishes an anisotropic nonlinear damage model in strain space to describe the behavior of jointed rock and applies it to mechanical analysis of tunnelling. This work focuses on rate-independent and small-deformation behavior during static isothermal processes. The prime results include: (1) the properties of damage-dependent elasticity tensors based on geological information of the jointed rock mass; (2) the damage evolution law presented on the basis of thermodynamics and combined with endochronic theory; and (3) the global damage tensor based on the work-equivalence principle and the local geological data of multi-joint sets. Finally the numerical results of a tunnel intersection in jointed rock is presented to illustrate the mechanical behavior of this model. © *1998 Elsevier Science Ltd. All rights reserved*

INTRODUCTION

Jointed rock has complex mechanical behavior, such as anisotropy, hysteresis, dilatancy, irreversible strain and strongly path-dependent stress–strain relationships, which is generally associated with the existence of a great deal of cracks and their propagation. In recent years, much work has been performed on the description and calculation of anisotropic mechanical behavior caused by cracks in solids.

Micromechanical [1–4] approaches attempted to predict the macroscale thermomechanical response of heterogeneous materials based on mesostructural models of a representative volume element (RVE) within the

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material. Micromechanical models have the distinct advantage of being able to capture structure details at the microscale and mesoscale, and of permitting formulation of the kinetic equation for damage evolution based on the physical process involved. But these models can be computationally inefficient in many practical applications and can therefore only be applied to limited cases.

Continuum Damage Mechanics (CDM) [5–12] which employs some continuum variables to describe cracks and joints, provides another way of modelling jointed rock mass. CDM is based on the thermodynamics of irreversible process, internal state variable theory and relevant physical considerations. Formal CDM modelling of the cracked rock was first suggested by Kawamoto *et al.* [8]. Thereafter additional advances were reported in some simple cases of anisotropic damage analysis [13]. In spite of all the efforts described above, it has not been possible until now to directly use the anisotropic damage theory for jointed rock mass in the mechanical analysis of engineering rock mass.

In this paper, the phenomenological second-order tensor which is directly related to the geological data of the jointed rock mass is chosen as the damage tensor of CDM. The concept of the active damage tensor is introduced in order to reflect the phenomenon that the cracks in the rock mass may close in compression, which means the damage tensor may be changed by compression or tension of strain state but not its propagation. A one-parameter damage-dependent elasticity tensor is formulated by tensorial algebra and thermodynamic requirements. The damage evolution law is formulated in the conjugate force space based on the irreversible thermodynamics and endochronic theory. Numerical tests have been performed on a series of typical simple examples. The engineering application to tunnelling is presented in the end of the paper. It is shown that the suggested damage formulation is suitable for the description of jointed rock mass.

DAMAGE DEFINITION

During the development of the damage theory, the definition of the damage tensor was also changed from a scalar formulation to a higher-order tensor form.

Scalar form

Two versions of the scalar form exist:

(1) The original Kachanov's model [5] considers the damage as a scalar. Further modifications had been made by many others. Considering the

simple one-dimensional case of homogeneous damage, the definition of the scalar is given as the effective surface density of microdefects:

$$D = \frac{S_D}{S} \tag{1}$$

where S is the area of the cross section and S_D is the effective area of the microcrack lie in S.

(2) The double scalar damage model of Lemaitre [14] is given for anisotropic damage behavior of composite materials. This model defines two independent scalar variables which characterize the damage influencing the elastic shear energy and the elastic hydrostatic energy: D_s acts on the deviatoric stress components and D_n on the hydrostatic stress. If a plane stress condition is assumed in the (x_1, x_2) plane, and the damage does not affect the behavior in x_1 direction, then the complementary energy density, which is the central role in the elastic constitutive relation, must be written as:

$$w_e = \frac{1}{2}\left[\frac{\sigma_{11}^2}{E_1} + \frac{\sigma_{22}^2}{E_2(1-D_n)} - \left(\frac{\nu_{12}}{E_1} + \frac{\nu_{21}}{E_2}\right)\sigma_{11}\sigma_{22} + \frac{\sigma_{12}^2}{G_{12}(1-D_s)}\right] \tag{2}$$

where E, ν, and G are Young's modulus, Poisson's ratio and the shear modulus, respectively.

Vector form

A comprehensive thermomechanical model based on the vectorial representation of the damage was formulated by Davison and Stevens [6]. The damage is locally defined as:

$$\omega_j(x_i, t) = \omega(x_i, t)N_j \tag{3}$$

where the scalar ω is the crack density in a cross section defined by its normal N_j.

Second-order tensor definition of damage

(1) Murakami and Ohno [7] proposed a damage model for a body with some sets of plane cracks based on the geometrical description of the crack system:

$$\Omega = \mathbf{I} - \frac{\mathbf{S}^{\text{net}}}{\mathbf{S}} \tag{4}$$

$$S_i^{\text{net}} = \left(\delta_{ij} - \Omega_{ij}\right) S_j \tag{5}$$

where $\mathbf{S}^{net}$ or S_i^{net} is the undamaged area vector in any cross section of the material; S or S_i is the total area vector.

(2) Kawamoto *et al.* [8] presented a modification of the above model for jointed rock. The given damage model for one set of cracks is:

$$\Omega^{\mathbf{k}} = \frac{l}{V} a^k \left(\mathbf{n}^{\mathbf{k}} \mathbf{n}^{\mathbf{k}}\right) \tag{6}$$

where a^k is the size of the cracked area and $\mathbf{n}^{\mathbf{k}}$ is the orientation vector of the crack.

For N-set of crack, the damage tensor was given as the sum over all the related damage tensor obtained with the above equation:

$$\Omega = \frac{l}{V} \sum_{k=1}^{N} a^k \left(\mathbf{n}^{\mathbf{k}} \mathbf{n}^{\mathbf{k}}\right) \tag{7}$$

(3) Dragon and Mroz [15] established their damage models for rock and concrete as follows:

$$\Omega_{ij} = \frac{1}{Vr} \sum_{k=1}^{m} \int n_i^{(k)} b_j^{(k)} \mathrm{dS}_{(k)} \tag{8}$$

where $n_i^{(k)}$ denotes the unit normal vector to the crack surface $S_{(k)}$ and $b_j^{(k)}$ denotes the displacement discontinuity (crack opening or sliding) at the crack surface in the representative element of volume V^r.

(4) On the basis of the micromechanical description of the crack sets and the consideration that all the cracks can be equivalently expressed with three mutually perpendicular families of parallel mesocracks Halm and Dragon [3] developed a damage model, which is also known as *fabric tensor*:

$$\Omega_{ij} = \frac{1}{V} \sum_{k=1}^{N} r_k^3 n_i^k n_j^k \tag{9}$$

where r_k is the average crack radii of one mesocrack family and $n^{(k)}$ is the normal direction of the crack set.

Other definition of damage

Based on the mechanical consideration that the damage tensor is the degradation of the elasticity tensor, Ju [16] defined a fourth-order damage tensor as follows:

$$\Omega_{ijkl} = I_{ijkl} - \frac{D_{ijmn}}{D^0_{mnkl}} \tag{10}$$

where D_{ijkl} and D^0_{ijkl} are damaged and undamaged elastic tensors, respectively. Since damage tensor Ω_{ijkl} possesses a one-to-one correspondence with the damage elastic tensor D_{ijkl}, D_{ijkl} is defined as an equivalent damage tensor.

Proposed damage definition

All the damage models described above can not be conveniently used in engineering calculations of jointed rock mass without any further improvement. The difficulties come either from calculation of the local damage tensor for one set of cracks, or from the global tensor, which should not be based on a simple summation over the entire local damage tensor.

The damage model adopted here is a second-order tensor. Its definition for one set of parallel joints is given in the following equation:

$$\Omega_{ij} = \omega n_i n_j, i = 1, 3 \tag{11}$$

where $\mathbf{n} = [l, m, n]^T$ is the direction vector of damage tensor Ω of the crack set; and ω is the separation factor of one set of cracks, which is taken as the crack density. The separation factor is part of the geological information of the jointed rock mass.

For a multiple set of joints, the global tensor should be calculated according to the work-equivalence principle. The details of its calculation will be presented in the following section.

DAMAGE ELASTICITY

Model of damage-dependent elasticity tensor

This subsection presents the development of an explicit expression of a damage-dependent elasticity tensor $\mathbf{D} = \mathbf{D}\,(\Omega)$. In general, an elasticity tensor is subject to the following general principles of continuum mechanics [17]:

(1) *Symmetry condition* requires $D_{ijkl} = D_{jikl} = D_{ijlk} = D_{klij}$;
(2) *Positive definite condition* requires the elastic potential function $W = \frac{1}{2}\varepsilon_{ij} D_{ijkl} \varepsilon_{kl}$ is positive-definite as a function of strain ε_{ij};
(3) *Material symmetry condition* requires that $\mathbf{D}(\Omega)$ be an isotropic tensor function. The isotropic tensor function $D_{ijkl}(\Omega_{mn})$, which satisfies the symmetric condition, takes the form

$$
\begin{aligned}
D_{ijkl} &= A_1\delta_{ij}\delta_{kl} + A_2\left(\delta_{ik}\delta_{jl} + \delta_{jk}\right)\\
&+ A_3\left(\Omega_{ij}\delta_{kl} + \Omega_{kl}\delta_{ij}\right) + A_4\left(\Omega_{ik}\delta_{jl} + \Omega_{il}\delta_{jk} + \Omega_{jk}\delta_{il} + \Omega_{jl}\delta_{ik}\right)\\
&+ A_5\Omega_{ij}\Omega_{kl} + A_6\left(\Omega_{ik}\Omega_{jl} + \Omega_{il}\Omega_{jk}\right)\\
&+ A_7\left(\Theta_{ij}\delta_{kl} + \Theta_{kl}\delta_{ij}\right) + A_8\left(\Theta_{ik}\delta_{jl} + \Theta_{il}\delta_{jk} + \Theta_{jk}\delta_{il} + \Theta_{jl}\delta_{ik}\right)\\
&+ A_7\left(\Theta_{ij}\delta_{kl} + \Theta_{kl}\delta_{ij}\right) + A_8\left(\Theta_{il}\delta_{jl} + \Theta_{il}\delta_{jk} + \Theta_{jk}\delta_{il} + \Theta_{jl}\delta_{ik}\right)\\
&+ A_9\left(\Theta_{ij}\delta_{kl} + \Omega_{ij}\Theta_{kl}\right) + A_{10}\left(\Theta_{ik}\Omega_{jl} + \Omega_{ik}\Theta_{jl} + \Theta_{il}\Omega_{jk} + \Omega_{il}\Theta_{jk}\right)\\
&+ A_{11}\Theta_{ij}\Theta_{kl} + A_{12}\left(\Theta_{ik}\Theta_{jl} + \Theta_{il}\Theta_{jk}\right)
\end{aligned}
\tag{12}
$$

where $\Theta_{ij} = \Omega_{im}\Omega_{mj}$; A_1, A_2, ..., A_{12} are functions of invariants of Ω_{ij} [18]. All existing expressions of the damage elasticity are special cases of Eqn (12). Enforcement the requirements on $A_1, A_2, ..., A_{12}$, as Cowin [19] suggested, makes the expression too complicated to use. Here, a one-parameter damage elasticity tensor is introduced:

$$\mathbf{D} = \phi \cdot \mathbf{D}^0 \cdot \theta \tag{13}$$

$$\tilde{D}_{ijkl} = \lambda\phi_{ij}\phi_{kl} + \mu\left(\phi_{ik}\phi_{jl} + \phi_{il}\phi_{jk}\right) \tag{14}$$

$$\phi_{ij} = \delta_{ij} - m\tilde{\Omega}_{ij} - (1-m)\tilde{\Omega}_{im}\tilde{\Omega}_{mj}, \quad 0 \le m \le 1 \tag{15}$$

where m is assumed to be the only damage-related material constant and λ and μ are the Lame's constants of the undisturbed rock mass. For jointed rock m is related to the crack system and is a function of the crack layer thickness. The condition $0 \le m \le 1$ is needed to satisfy the positive definite requirement. Evidently, the elasticity tensor satisfies all requirements and a complete tensorial polynomial as compared with Eqn (12).

To describe the closing and opening of the joint system in rock, an active damage tensor is introduced in the following subsection.

Active damage tensor

In rock-like material, the damage appears in the form of planar cracks. The cracks may close in compression and open in tension. In order to reflect this character of rock damage, the concept of active damage tensor is introduced here. By decomposing one original crack vector in the principal strain coordinate system, it is assumed that each equivalent crack is composed of only these components in tensile principal strain directions. Thus, the active damage tensor can be calculated by the transformation

$$\tilde{\Omega} = \mathbf{P}^{+} : \Omega \tag{16}$$

where $\mathbf{P}^{+}$ is just the so-called "positive projection tensor" [20],

$$P^{+}_{ijkl} = Q^{+}_{ik} Q^{+}_{jl}, \text{ where} \tag{17}$$

$$\mathbf{Q}^{+} = \sum_{i=1}^{3} \hat{H}(\varepsilon_i)\mathbf{p}_i\mathbf{p}_j$$

where Q^{+}_{jl} is the positive (tensile) spectral projection tensor, $\mathbf{p}_i$, are the ith principal direction vectors of strain tensor ε and ε_i are the principal strain values. $\hat{H}(\cdot)$ is the Heaviside function

$$\hat{H}(\varepsilon_i) = \begin{cases} 1 & \text{if } \varepsilon_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

The active damage tensor is symmetric, ensured by Eqn (16) and Eqn (17). In some geomaterials, the Heaviside function $\hat{H}$ will overestimate the difference between compression and tension. For example, in jointed rock, the joints are not perfectly in contact but contain some fillingmaterials or roughness. In order to take these factors into account, the Heaviside function $\hat{H}$ in Eqn (17) can be replaced with

$$H(\varepsilon_i) = (1-h)\hat{H} + h = \begin{cases} 1 & \text{if } \varepsilon_i > 0 \\ h & \text{otherwise} \end{cases} \tag{19}$$

where $h(0 \le h < 1)$ is a material constant to reflect the properties of crack contact.

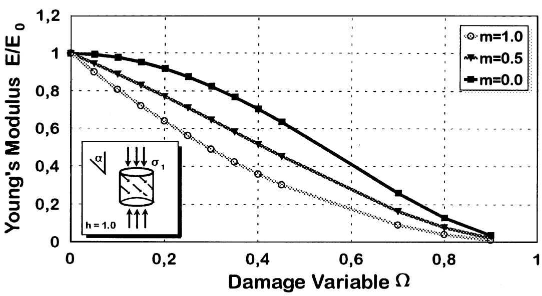
Numerical test of damage elasticity

The behavior of the damage-dependent elasticity tensor defined in Eqn (15) is illustrated by a series of numerical uniaxial tests. The cylinder contains a set of parallel microcracks. These parallel microcracks in the specimen can be characterized by a damage vector (n_i, ω), where ω is the characteristic damage value, and the normal vector n_i is determined by α, the angle between loading direction and the crack plane. The damage vector corresponds to a damage tensor $\omega n_i n_j$. With the increase in the damage value, the reduction of the normalized Young's modulus E/E_0 in the loading direction for different values of m is shown in Fig. 1(a) under the condition of $h = 1$. The results indicate that an increase in m will reduce the normalized module E/E_0.

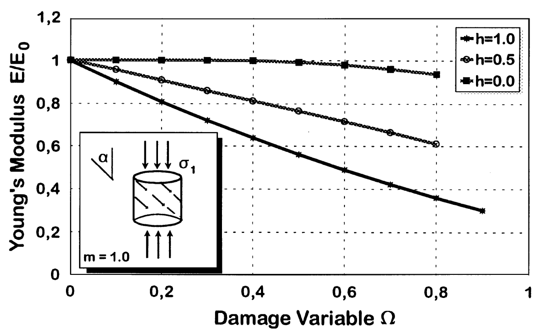
The relation between the normalized Young's modulus E/E_0 in the loading direction and the increment of damage value is shown in Fig. 1(b) based on the damage elasticity parameter $m = 1$. It is shown that if no residual damage effect is considered, meaning $h = 0$, there is no reduction of Young's modulus in the direction of compression. The stiffness of the material between the cracks can produce such an effect and would compensate the damage effect. With an assumption of $h = 1$, the active damage tensor will be equal to the nominal damage tensor for both tension and compression.

The relation between the apparent Young's modulus E/E_0 along the loading direction and the crack angle α is shown in Fig. 2(a). It indicates that the horizontal component of damage tensor exerts little influence on the apparent Young's modulus in the vertical direction under the action of uni-axial compression.

To study the influence of the lateral pressure on the stiffness in vertical direction a numerical simulation with 12 20-node isoparametric elements was



(a)



(b)

Fig. 1. (a) Effect of damage parameter m and (b) parameter h.

performed. On the cylinder act a uniform pressure σ_{top} on the top surface and lateral pressure σ_{lateral}. Using these two stresses the dimensionless factor $r = \sigma_{\text{lateral}}/\sigma_{\text{top}}$ can be defined. In the example of Fig. 2(b), three sets of joints are assumed to be present in the cylinder in order to simulate the engineering situation. The initial damage tensor Ω is obtained through the geological data ($\omega_1 = 0.2$, $\omega_2 = 0.6$, $\omega_3 = 0.3$). The definition will be shown in the next section.

The results for the $\sigma_z - \varepsilon_z$ diagram with different values of r are shown in Fig. 2(b). Reduction of the stiffness in z direction is not very seriously influenced by the lateral pressure.

Determination of damage tensor using the geological data of a joint set

The orientation of the joint set is usually identified in the field by sampling alone, a line, as in a borehole, or over an area, as on an outcropping surface. Statistical descriptions of the orientation data are always expressed as the orientation of the normal line of the joint plane. Spherical coordinates

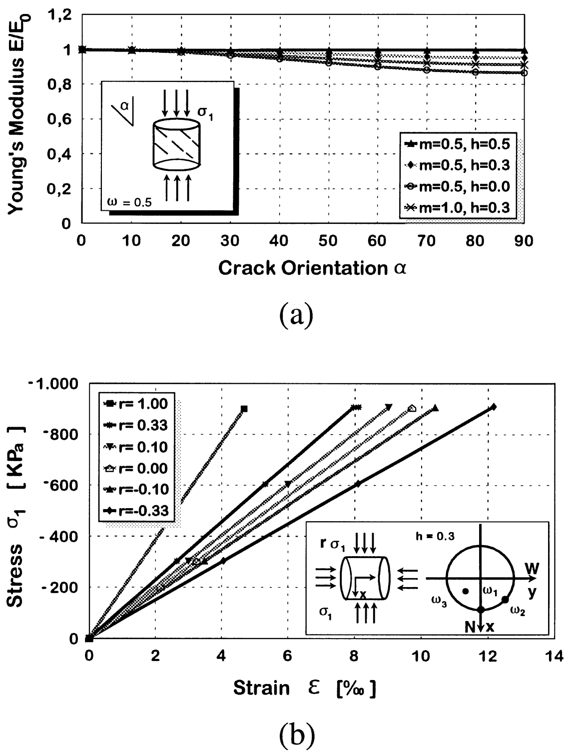


Fig. 2. (a) Effect of crack orientation; (b) influence of the lateral pressure.

(α, β, r) or the direction cosine (l, m, n) are generally used for this purpose (Fig. 3). These are related to the dip angle, which is defined for the angle between the horizontal and the line of maximum dip of the joint plane, β and the dip direction, which is defined for the angle between north and the horizontal projection of the line of maximum dip of the joint plane measured in clockwise direction, α, together with the direction angle of the tunnel axis γ, the angle between the tunnel axis and north, as follows:

$$\begin{cases} l = \sin\beta\cos[2\pi - (\alpha+\gamma)] \\ m = \sin\beta\sin[2\pi - (\alpha+\gamma)] \\ n = \cos\beta \end{cases} \tag{20}$$

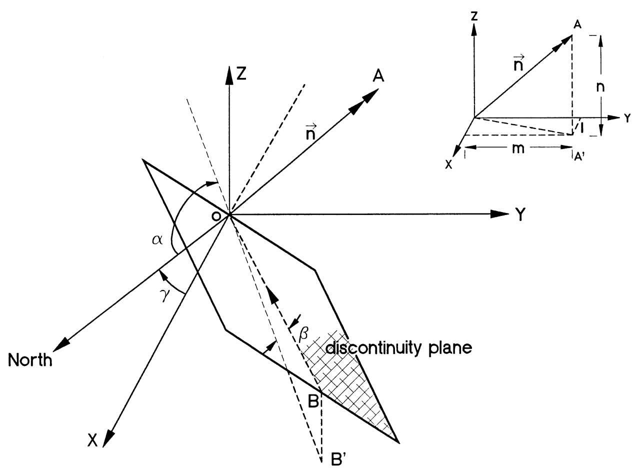


Fig. 3. Relationship between orientation and the spherical coordinate system.

The direction vector of the damage tensor Ω according to Eqn (1) and Eqn (2) is defined on the basis of the direction vector $\mathbf{n} = [l, m, n]^T$. Here an example for calculating the damage tensor from the geological data is given. The dip direction $\alpha = 45°$, dip angle $\beta = 45°$, tunnel axis direction angle $\gamma = 0$, damage parameter $\omega = 0.3$ are based on the geological investigation. Using the above-mentioned calculation process, the initial damage tensor would be:

$$\Omega_0 = \begin{bmatrix} 0.075 & -0.075 & 106 \\ -0.075 & 0.075 & -0.106 \\ 0.106 & -0.106 & 0.15 \end{bmatrix}$$

Global damage tensor for multiple sets of joints

For jointed rock mass, the most common damage state is the presence of multiple sets of joints which intersect each other. Presented here are the techniques for determining the global damage tensor which can represent the damage effect of the multiple joint sets system. The damage tensor related to each set of joints can be obtained with the techniques introduced above. Calculation of the global damage tensor is based on the principle of energy equivalence: The energy dissipated by the global damage tensor is equivalent to the sum of the energy dissipated by the damage tensor related to each set of joints.

The equation for calculating the global damage tensor is given in the following:

$$\Omega_g = \mathbf{I} - \left[\sum_{n=1}^{N} \left(\mathbf{I} - {}_n \right)^{-1} - (N-1)\mathbf{I} \right]^{-1} \tag{21}$$

where ${}_g$ is the global damage tensor; ${}_n$ is the damage tensor for the nth joint; N is the total number of joint sets; $\mathbf{I}$ is a second-order unit tensor.

Calculation of the global damage tensor is shown for three sets of joints whose geological data are:

$$\begin{array}{lll} \gamma = 0, & & \\ \alpha_1 = 0, & \beta_1 = \frac{\pi}{2}, & \omega_1 = 0.2 \\ \alpha_2 = -\frac{\pi}{4}, & \beta_2 = \frac{\pi}{2}, & \omega_2 = 0.6 \\ \alpha_3 = \frac{\pi}{4}, & \beta_3 = \frac{\pi}{4}, & \omega_3 = 0.3 \end{array}$$

Thus, the damage tensor for each set of joints is:

$$\begin{array}{l} \Omega_1 = \begin{bmatrix} 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \\ \Omega_2 = \begin{bmatrix} 0.3 & 0.3 & 0.0 \\ 0.3 & 0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \Omega_3 = \begin{bmatrix} 0.075 & -0.075 & 0.106 \\ -0.075 & 0.075 & -0.106 \\ 0.106 & -0.106 & 0.15 \end{bmatrix} \end{array} \tag{22}$$

The global damage tensor for the three joint sets, according to Eqn (21), is:

$$
\Omega = \begin{bmatrix} 0.45946 & 0.19459 & 0.01 \\ 0.19459 & 0.38595 & -0.1 \\ 0.01 & -0.1 & 0.15243 \end{bmatrix}
$$

DAMAGE EVOLUTION

Damage evolution law is one of the most important aspects of the constitutive relationship of rock. In this section it is assumed that the damage evolution is completely determined by its conjugate force, which is also known as *the strain energy density release rate* [2]. Using the principle of endochronic theory [21, 22], this evolution law for the isothermal static damage problems is established in the framework of thermodynamics. No softening or hardening related to damage process is considered.

Description of damage evolution law

Firstly, following the form given in [22], the potential function ϕ for damage evolution can be expressed as follows:

$$
\phi^2 = \frac{1}{2} R_{ij} B_{ijkl} R_{kl}, \quad B_{ijkl} = B I_{ijkl} \tag{23}
$$

where R_{ij} is the conjugate force tensor, I_{ijkl} is the fourth-order unit tensor and B is a material constant governing the behavior of damage propagation.

According to the theory of thermodynamics the damage propagation $\dot{\Omega}_{ij}$ is obtained from the relation:

$$
\dot{\Omega}_{ij} = \dot{\lambda} \frac{\partial \phi}{\partial R_{ij}} = \dot{\lambda} \frac{B_{ijkl}}{2\phi} R_{kl} \tag{24}
$$

where $\dot{\lambda}$ is a multiplier.

Next, endochronic theory is introduced into the calculation of damage evolution. Endochronic theory assumes that the ratio of an internal variable is proportional to its conjugate force. Based on this principle the damage evolution in the intrinsic time space is obtained:

$$
\frac{\mathrm{d}\Omega}{\mathrm{d}z} = C_{ijkl} R_{kl} \tag{25}
$$

where C_{ijkl} is a constant coefficient tensor; and $\mathrm{d}z$ is the infinitesimal increment of intrinsic time scale defined in elastic strain ε^e_{ij} space:

$$\mathrm{d}z = \sqrt{\mathrm{d}e_{ij}\mathrm{d}e_{ij}},\ e_{ij} = \varepsilon_{ij}^{e} - \frac{1}{3}\delta_{ij}\varepsilon_{e}^{kk} \tag{26}$$

The following relation is obtained by comparing Eqn (24) and Eqn (25):

$$C_{ijkl} = \dot{\lambda}\frac{B_{ijkl}}{2\phi} \tag{27}$$

Replacing Newton time with the intrinsic time dz, the damage evolution equation can be written as follows:

$$\frac{\mathrm{d}\Omega}{\mathrm{d}z} = \dot{\lambda}\frac{B_{ijkl}}{2\phi}R_{kl} \tag{28}$$

Damage propagation is controlled through the threshold value of the damage potential ϕ^0, which is the second parameter for damage evolution.

The meaning of $\dot{\lambda}$ is defined as follows:

$$\dot{\lambda} = \begin{cases} 1 & \text{if}\phi > \phi^0; \quad \text{propagation} \\ 0 & \text{if}\phi \le \phi^0; \quad \text{elastic} \end{cases} \tag{29}$$

Therefore, the equation for an infinitesimal increment of the damage tensor is obtained:

$$d\Omega = \begin{cases} \frac{B_{ijkl}}{2\phi}R_k\mathrm{d}z & \text{if } \phi > \phi^0 \\ 0 & \text{if } \phi \le \phi^0 \end{cases} \tag{30}$$

For a given loading process, the calculation of the conjugate force R_{ij} is:

$$R_{ij} = P_{ijkl}^{+}\tilde{R}_{kl} = P_{ijkl}^{+}\left(\frac{1}{2}\varepsilon_{ij}^{e}\frac{\partial D_{ijkl}}{\partial\Omega_{mn}}\varepsilon_{kl}^{e}\right) \tag{31}$$

Making integration over Eqn (34) can provide the increment in the damage tensor $\Delta\Omega_{ij}$ corresponding to a load increment.

Finite element implementation of the damage evolution law

In order to apply the theory described above to solve the nonlinear elastic damage problems, this section presents the following numerical solution scheme, which is combined with the theory of nonlinear finite element methods [23].

Corresponding to a load increment, for the nth iteration step the system equation is:

$$\mathbf{K}_n \Delta a = \mathbf{f}_{\mathbf{n}}^r \tag{32}$$

with

$$\mathbf{K}_n = \int_V \mathbf{B}^T \mathbf{D}_n \mathbf{B} \mathrm{d}V, \mathbf{f}^r = \int_V \mathbf{B}^T \sigma^r \mathrm{d}V \tag{33}$$

where $\mathbf{K}_n$ is the system stiffness matrix, $\Delta\mathbf{a}$ the vector of displacement increment, and $\mathbf{f}_{\mathrm{n}}^r$ the vector of unbalanced force of this iteration step. The Newton–Raphson method, which is also known as the tangential stiffness method, is adopted in the solution presented for Eqn (32). The damage increment $\Delta\Omega_{ij}$, the total stress σ_n and residual stress σ^r can be respectively calculated as follows:

$$\Delta\Omega = \begin{cases} \int_{z1}^{z2} \frac{B_{ijkl}}{2\phi} R_{kl} \mathrm{d}z & \text{if } \phi > \phi^0 \\ 0 & \text{if } \phi \le \phi^0 \end{cases} \tag{34}$$

$$\sigma_n = \mathbf{D}_n \varepsilon_n + g\Delta\Omega \tag{35}$$

$$\sigma^r = \sigma_{n-1} - \mathbf{D}_n \varepsilon_{n-1} - g\Delta\Omega \tag{36}$$

Numerical test of damage evolution

The application of the theory is shown with the help of the simple supported beam examples. The behavior of the model is tested under the assumption that we have one set of joints with a dip direction: $\alpha = 0°$, $0 = 45°$, the separation factor $\omega_0 = 0.5$.

The model of the beam is shown in Fig. 4. The external loading F acting in the middle section is incrementally applied. As shown in Fig. 4(a), with the increase in load, the crack starts to propagate and the active damage value becomes larger than the initial value. The unsymmetrical distribution of the active damage shown in Fig. 4(b) shows the impact of crack orientation. The influence of the values of damage energy threshold $\phi°$ is shown in Fig. 4(c). With a higher value of $\phi°$ the effective stiffness becomes higher, corresponding to certain loading values. The influence of the values of constant B in

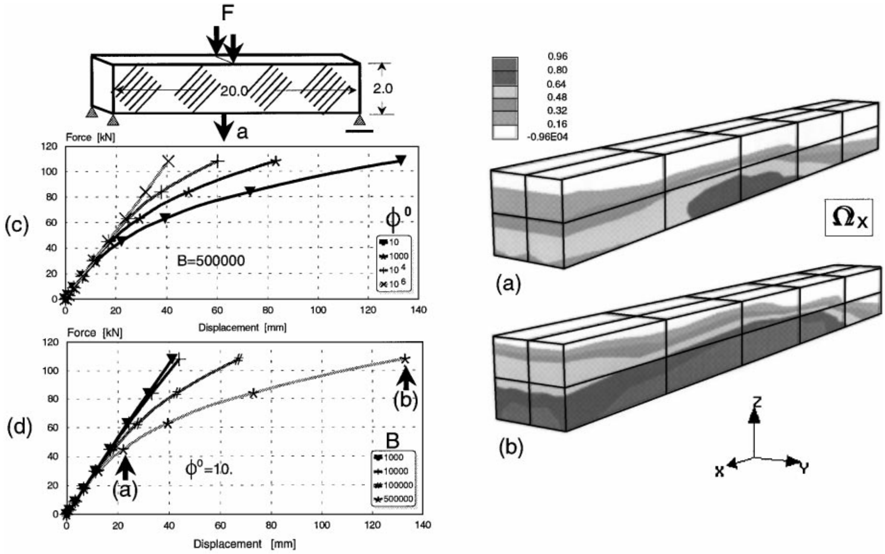


Fig. 4. Simply supported beam model: damage evolution.

Eqn (23) is shown in Fig. 4(d). With a higher value of B the effective stiffness becomes higher, corresponding to certain loading values, which means the relevant active damage tensor is smaller.

APPLICATION TO A TUNNEL INTERSECTION IN JOINTED ROCK

The 3-km-long Schönberg Tunnel was newly built for the by-pass around the city of Schwarzach (Salzburg). The tunnel had to be constructed in heavily jointed phyllite rock (Fig. 5). The construction was a combination of a TBM pilot tunnel and NATM for final construction. For the intersection between the main tunnel and one of the escape tunnels a three-dimensional model was used to simulate the impact of the jointed rock on the construction. The parameters of therock mass are:

$$E_1 = 20.0\text{MPa}, \quad \mu_1 = 0.25 \tag{37}$$

and that of the shotcrete lining are:

$$E_2 = 20.0\text{MPa}, \quad \mu_2 = 0.2 \tag{38}$$

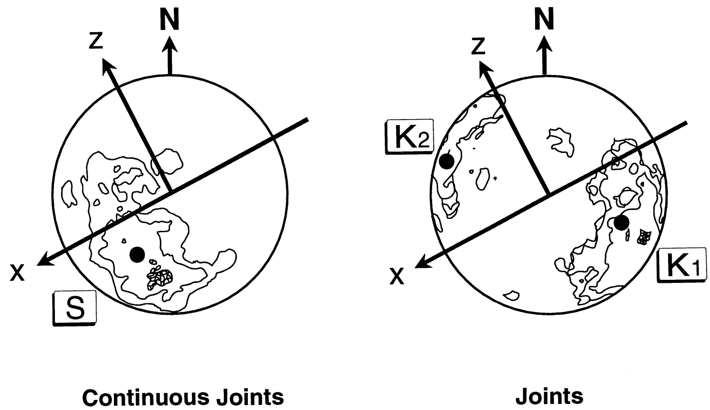


Fig. 5. Geological statistics of the joint set S, $K1$, $K2$.

In the numerical model it was very important to very carefully simulate all construction steps from the pilot tunnel, to the crown excavation, to the escape tunnel and the final lining. The intersection was analyzed with two systems of jointed rock. The first joint system was a set of joints with the following orientation and initial damage:

$$\alpha = 90^\circ, \beta = 45^\circ, \gamma = 180^\circ, \omega = 0.5 \tag{39}$$

Due to the initial ground stress all cracks are closed, which results in an active damage tensor, namely zero. Already after excavation of the pilot tunnel, as shown in Fig. 6, the cracks start to open, presented by the active damage tensor. The cracks open mainly in this part of the tunnel, where they are parallel to the tunnel surface. During construction of the main tunnel the damage region increases in the same direction, as shown in Fig. 7. The result of excavation of both tunnels is shown in Fig. 8, which shows that rock loosening appears in a great area around the tunnel. Damage propagation was not very serious; the maximum principal damage value increased from 0.5 to a value of 0.58. It is important to mention that a conventional elastoplastic calculation with the material parameters used above and the Mohr–Coulomb, yielding criteria did not show any plastic region at all.

From the geological data surveyed after construction of the pilot tunnel, it was found that this area contains a continuous joint system S, as shown in Fig. 5, and two additional joint sets K1 and K2. The initial damage data of the joint sets was estimated by the geological expert as follows:

$$\omega_S = 0.9, \omega_{K1} = 0.5, \omega_{K2} = 0.4 \tag{40}$$

The statistical data of these joint sets are shown in Fig. 5. Using the orientation of the real joint system and the orientation of the coordinate system adopted in this calculation, the final initial damage tensor can be determined. The final result of the principal active damage is shown in Fig. 9(a) before damage propagation and in Fig. 9(b) after damage propagation. The impact of new cracks is very small. The maximum principal damage increases from the initial value 0.9 to 0.99. It is also of great interest that from a practical point of view, the maximum damage is not around the main tunnel; it is found around the small tunnel. But also the maximum damage in the main tunnel is not close to the lining; it is found at some distance from the excavation. This is well known by tunnel engineers; it is the so-called support ring found in the rock around the tunnel. The active damage zone is also very important for the tunnel engineer, namely as a support for laying out the anchoring system.

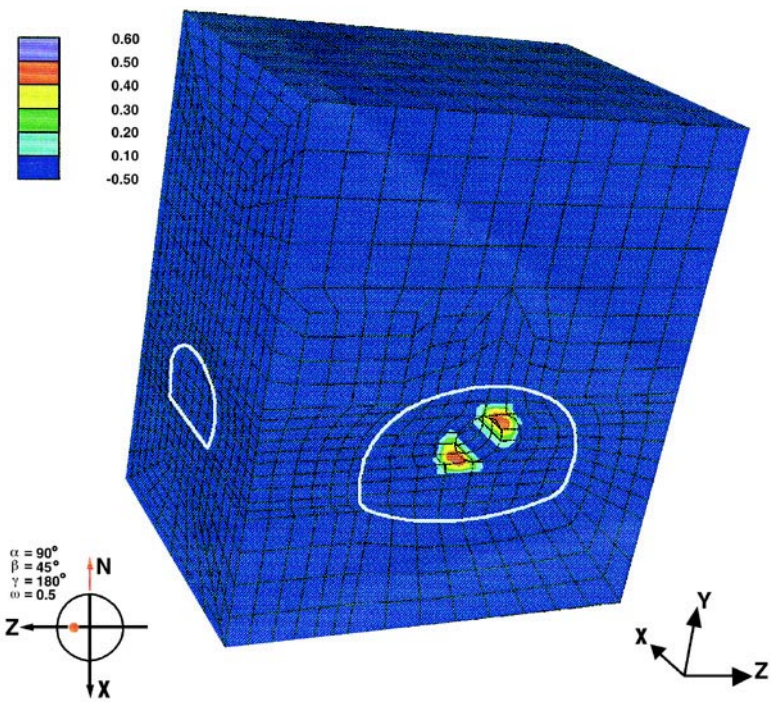


Fig. 6. Distribution of maximum principal active damage: loading 1.

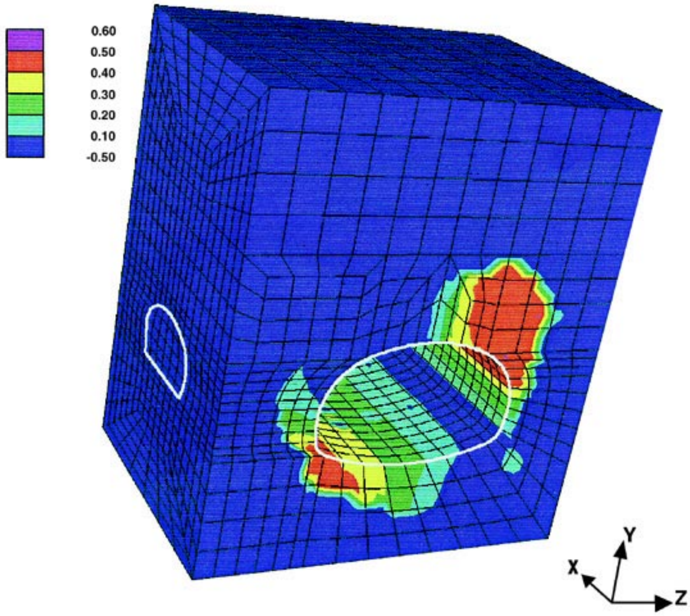


Fig. 7. Distribution of maximum principal active damage: loading 7.

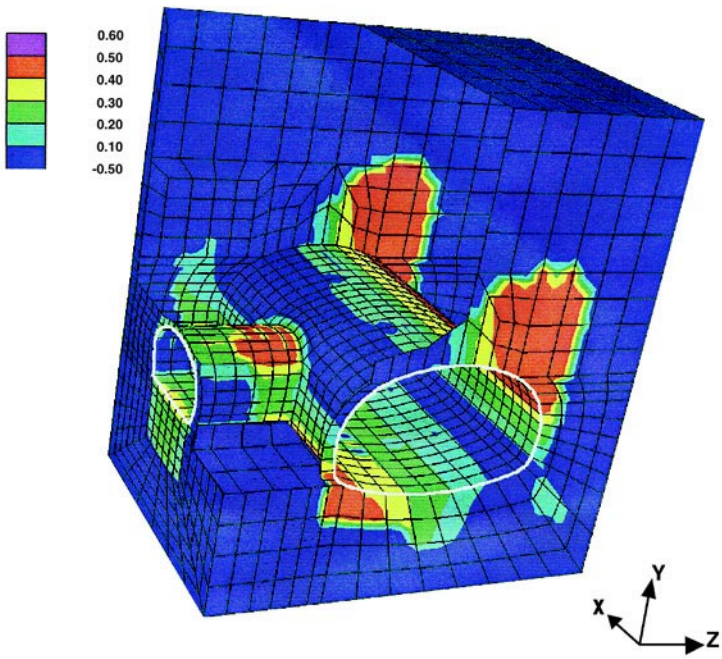


Fig. 8. Distribution of maximum principal active damage: loading 10.

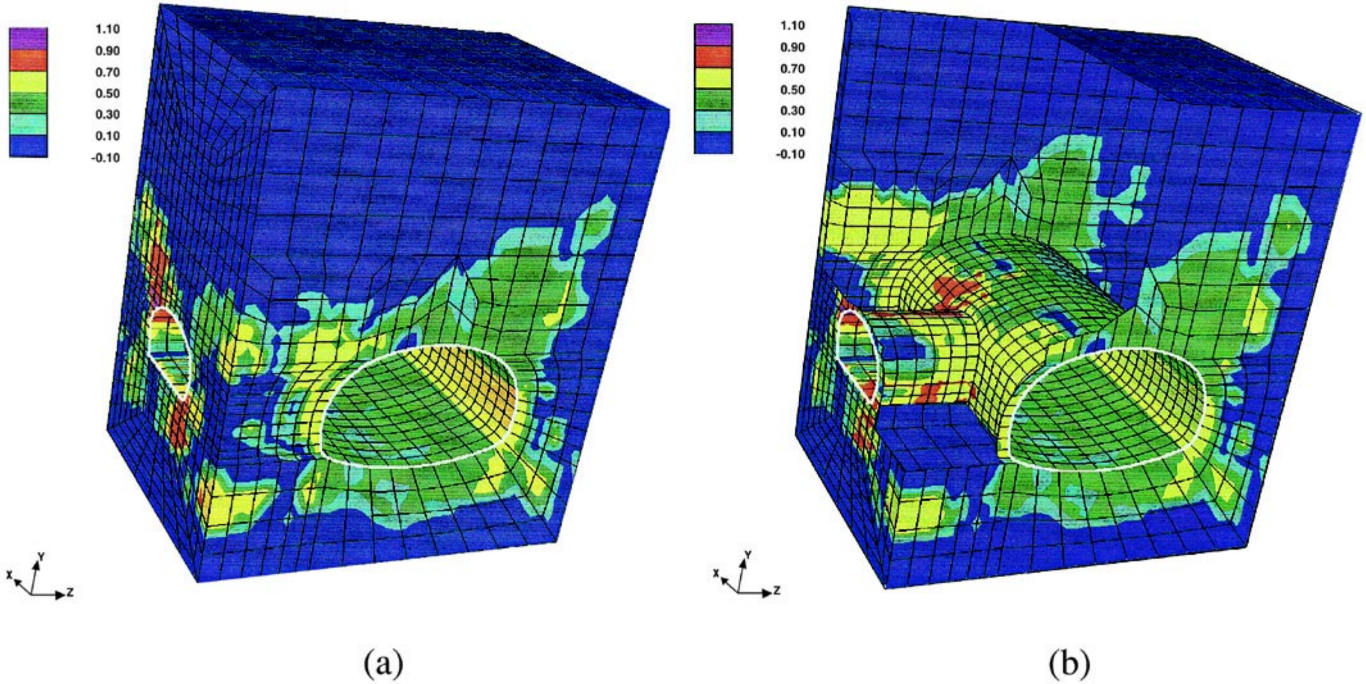


Fig. 9. Damage for 3 joint sets (a) before and (b) after damage: propagation: loading 10.

CONCLUSION

Determination of the initial damage tensor of rock mass is one of the principal difficulties in the applying CDM to geoengineering. In this paper the initial damage tensor of rock mass is determined on the basis of the geological data of joint sets obtained by *in situ* measurements. The influence of closure of cracks under compression is considered by adopting the concept of *active damage tensor*.

Degradation of the elasticity tensor resulting from damage evolution was investigated with the Finite Element Methods. Parameter sensitivity was also studied numerically.

The damage evolution law was presented on the basis of thermodynamics and the endochronic theory. Numerical scheme for solving nonlinear damage problems is also given.

A simple example of a typical mechanical model is investigated numerically in order to demonstrate the validity of the above damage theory. The results coincide with the experimental results presented in the reference literature.

The damage theory presented in this paper was applied in the mechanical analysis of the Schönberg Tunnel in Austria. The numerical results of damage distribution are reasonable and coincide well with *in situ* engineering experience.

APPENDIX

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