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CONSTITUTIVE THEORY OF PLASTICITY COUPLED WITH ORTHOTROPIC DAMAGE FOR GEOMATERIALS*

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Abstract: Constitutive theory of plasticity coupled with orthotropic damage for geomaterials was established in the framework of irreversible thermodynamics. Prime results include: 1) evolution laws are presented for coupled evolution of plasticity and orthotropic damage; 2) the orthotropic damage tensor is introduced into the Mohr-Coulomb criterion through homogenization. Both the degradation of shear strength and degradation of friction angle caused by damage are included in this model. The dilatancy is calculated with the so-called damage strain.

Key words: damage; plasticity; coupling; dilatancy; geomaterial

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Introduction

For geomaterials such as rock and concrete, the frictional sliding along microcrack surfaces is associated with the plasticity model described by the Mohr-Coulomb theory; moreover crack propagation corresponds to damage growth. The coupling of damage and plasticity of a phenomenological model has two meanings. Firstly, the coupling means that damage and plasticity affect each other through their potential function. Secondly, it means that the respective consistency condition should be satisfied simultaneously. In other words, the evolution of damage and plastic internal variable are coupled.

It is convenient to consider the strength degradation caused by damage through the concept of effective stress and develop the plastic flow theory in the effective stress space, as it was described by numerous researchers[1-4]. However this approach can not perfectly account for the

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effects of damage on the behavior of frictional sliding of geomaterails. In order to consider in detail the behavior of frictional sliding, Basista and Gross[5] and Dragon and Halm[6] have respectively presented their sliding crack model on the basis of internal variable theory. The single flow surface model for coupled damage-plasticity had also presented by some researchers[7],[8].

In engineering practice, experimental results indicated that the relationship between compressive lateral normal traction and the critical shear strength is nonlinear, which is not in accordance with the Mohr-Coulomb criteria. Numerous researchers worked on this topic from different points of view[9]. Several different versions of Mohr-Coulomb type criteria were proposed for critical criteria of shear failure for geomaterials[10],[11].

In this study, an orthotropic phenomenological model of plastic degradation coupled with damage is presented for geomaterials on the basis of thermodynamics and physical background of microstructure. Both the degradation of the shear strength and variation of the angle of frictional are considered in this model. The actual stress space is used for the calculation of plasticity and damage. Damage is introduced into the Mohr-Coulomb criteria on the basis of homogenization theory[12] instead of using the concept of effective stress. No additional material parameters are introduced in this model except those already adopted in plasticity and damage theory.

1 General Thermodynamic Framework

1.1 Continuum thermodynamics

Let us restate briefly the thermodynamic structure of constitutive models of damage and plasticity, and introduce the thermodynamic potentials expressed in terms of macroscopic variables. Consider first the specific internal state energy $u = u[x, s(x), \boldsymbol{\varepsilon}(x), \boldsymbol{\xi}_i(x)]$, per unit mass, where $\boldsymbol{\varepsilon}$ and s denote the strain and entropy respectively; $\boldsymbol{\xi}_i$ is the set of internal state variables. The Clausius-Duhem inequality can be expresses as follows:

$$\rho\theta\dot{s} \geq \rho\dot{u}(s, \boldsymbol{\varepsilon}, \boldsymbol{\xi}_i) - \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}, \tag{1}$$

where $\boldsymbol{\rho}$ denotes the material density, $\boldsymbol{\theta}$ is the absolute temperature, and $\dot{s}_i$ is the irreversible entropy production. The dot over a symbol denotes rate or increment. From Eq. (1), we obtain the inequality:

$$\rho\left(\theta - \frac{\partial u}{\partial s}\right)\dot{s} + \left(\boldsymbol{\sigma} - \rho\frac{\partial u}{\partial \boldsymbol{\varepsilon}}\right):\dot{\boldsymbol{\varepsilon}} - \rho\frac{\partial u}{\partial \boldsymbol{\xi}_i}\dot{\boldsymbol{\xi}}_i \geq 0. \tag{2}$$

For the reversible process, there is $\dot{\boldsymbol{\xi}}_i = 0$ and $\rho\theta\dot{s} = 0$. Hence for arbitrary rates $\dot{\boldsymbol{\varepsilon}}_i$ and $\dot{s}$, the following potential relation occurs:

$$\boldsymbol{\sigma} = \rho\frac{\partial u}{\partial \boldsymbol{\varepsilon}}, \quad \theta = \frac{\partial u}{\partial s}, \tag{3}$$

and dissipation rate equals:

$$Y = -\rho\frac{\partial u}{\partial \boldsymbol{\xi}_i}\dot{\boldsymbol{\xi}}_i. \tag{4}$$

For constitutive modeling, more useful are dual thermodynamic potentials, namely the free strain energy:

$$\boldsymbol{\Phi} = u - \theta s = \boldsymbol{\Phi}(\boldsymbol{\varepsilon}, \theta, \boldsymbol{\xi}_i), \tag{5}$$

and the free stress energy:

$$\boldsymbol{\Psi} = \boldsymbol{\sigma}:\boldsymbol{\varepsilon} - \boldsymbol{\Phi} = \boldsymbol{\Psi}(\boldsymbol{\sigma}, \theta, \boldsymbol{\xi}_i). \tag{6}$$

For which the Clausius-Duhem inequality is expressed as follows :

$$Y = \left(\boldsymbol{\sigma} - \rho \frac{\partial \Phi}{\partial \boldsymbol{\varepsilon}} \right) : \dot{\boldsymbol{\varepsilon}} - \rho \left(s + \frac{\partial \Phi}{\partial \theta} \right) \dot{\theta} - \rho \frac{\partial \Phi}{\partial \boldsymbol{\xi}_i} \dot{\boldsymbol{\xi}}_i \geq 0. \qquad (7)$$

In another form :

$$Y = \left(\frac{\partial \Psi}{\partial \boldsymbol{\sigma}} \right) : \dot{\boldsymbol{\sigma}} + \rho \left(\frac{\partial \Psi}{\partial s} - s \right) \dot{s} + \rho \frac{\partial \Psi}{\partial \boldsymbol{\xi}_i} \dot{\boldsymbol{\xi}}_i \geq 0. \qquad (8)$$

The following constitutive relations follow Eq. (7) and Eq. (8)

$$\boldsymbol{\sigma} = \rho \frac{\partial \Phi}{\partial \boldsymbol{\varepsilon}}, \quad s = - \frac{\partial \Phi}{\partial \vartheta}, \quad Y_i = - \rho \frac{\partial \Phi}{\partial \boldsymbol{\xi}_i}, \qquad (9)$$

$$\boldsymbol{\varepsilon} = \rho \frac{\partial \Psi}{\partial \boldsymbol{\sigma}}, \quad s = \frac{\partial \Psi}{\partial \vartheta}, \quad Y_i = \rho \frac{\partial \Psi}{\partial \boldsymbol{\xi}}, \qquad (10)$$

where Y_i is the so-called conjugate force of the internal variable $\boldsymbol{\xi}_i$.

The incremental or rate-form equations for stress and strain can now be expressed as follows :

$$\dot{\boldsymbol{\sigma}} = \rho \frac{\partial^2 \Phi}{\partial \boldsymbol{\varepsilon} \partial \boldsymbol{\varepsilon}} : \dot{\boldsymbol{\varepsilon}} + \rho \frac{\partial^2 \Phi}{\partial \boldsymbol{\varepsilon} \partial \vartheta} \dot{\vartheta} + \rho \frac{\partial^2 \Phi}{\partial \boldsymbol{\varepsilon} \partial \boldsymbol{\xi}_i} : \dot{\boldsymbol{\xi}}_i, \qquad (11)$$

$$\dot{\boldsymbol{\varepsilon}} = \rho \frac{\partial^2 \Phi}{\partial \boldsymbol{\sigma} \partial \boldsymbol{\sigma}} : \dot{\boldsymbol{\sigma}} + \rho \frac{\partial^2 \Psi}{\partial \boldsymbol{\sigma} \partial \vartheta} \dot{\vartheta} + \rho \frac{\partial^2 \Psi}{\partial \boldsymbol{\sigma} \partial \boldsymbol{\xi}_i} : \dot{\boldsymbol{\xi}}_i. \qquad (12)$$

Consider an isothermal process $\dot{\theta} = 0$, and introduce plastic and damage variables $\boldsymbol{\xi}_p = P$, $\boldsymbol{\xi}_D = D$. Eqs. (11) and (12) can be rewritten in the form :

$$\dot{\boldsymbol{\sigma}} = \rho \frac{\partial^2 \Phi}{\partial \boldsymbol{\varepsilon} \partial \boldsymbol{\varepsilon}} : \dot{\boldsymbol{\varepsilon}} + \rho \frac{\partial^2 \Phi}{\partial \boldsymbol{\varepsilon} \partial P} : \dot{P} + \rho \frac{\partial^2 \Phi}{\partial \boldsymbol{\varepsilon} \partial D} : \dot{D} = \dot{\boldsymbol{\sigma}}^e + \dot{\boldsymbol{\sigma}}^P + \dot{\boldsymbol{\sigma}}^D, \qquad (11a)$$

$$\dot{\boldsymbol{\varepsilon}} = \rho \frac{\partial^2 \Psi}{\partial \boldsymbol{\sigma} \partial \boldsymbol{\sigma}} : \dot{\boldsymbol{\sigma}} + \rho \frac{\partial^2 \Psi}{\partial \boldsymbol{\sigma} \partial P} : \dot{P} + \rho \frac{\partial^2 \Psi}{\partial \boldsymbol{\sigma} \partial D} : \dot{D} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^P + \dot{\boldsymbol{\varepsilon}}^D. \qquad (12a)$$

The total stress and strain rates can be decomposed into elastic , plastic and damage components specified by the relation Eq. (11a) and Eq. (12a) . Assume , in particular , that :

$$\left. P = \boldsymbol{\varepsilon}^P, \quad \rho \Phi = W(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^P, D) = \frac{1}{2} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P) : E(D) : (\boldsymbol{\varepsilon}, - \boldsymbol{\varepsilon}^P), \right\} \qquad (13)$$

Then we have

$$Y_D = \frac{1}{2} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P) : \frac{\partial E(D)}{\partial D} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P), \quad Y_P = \boldsymbol{\sigma} = E(D) : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P), \qquad (14)$$

where $L(D) = E(D)^{-1}$ is the elastic compliance tensor. And dissipation rate is

$$Y = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^P + Y : \dot{D} \qquad (15)$$

1.2 Flow and damage rules

Assume the yield condition of Mohr-Coulomb type to be affected by damage tensor is

$$F^P = F^P(\boldsymbol{\sigma}, D, c^0, \varphi^0) \leq 0, \qquad (16)$$

where c^0, φ^0 are the initial cohesion and the friction angle. The damage condition is expressed in terms of the conjugate force of damage , thus :

$$F^D = F^D(Y, \boldsymbol{\varepsilon}^P, \omega^0) \leq 0. \qquad (17)$$

The plastic flow and damage evolution rules are expressed as follows :

$$\dot{\boldsymbol{\varepsilon}}^P = \lambda^P \frac{\partial F^P}{\partial \boldsymbol{\sigma}}, \quad \lambda^P > 0, \quad F^P = 0, \quad \lambda^P F^P = 0, \qquad (18)$$

$$\dot{D} = \lambda^{D} \frac{\partial F^{D}}{\partial Y_{i}}, \quad \lambda^{D} > 0, \quad F^{D} = 0, \quad \lambda^{D} F^{D} = 0. \tag{19}$$

Four regimes can be distinguished, namely:

1) Elastic regime, $F^{P} < 0$, $F^{D} < 0$;

2) Damage regimes, $F^{D} = \dot{F}^{D} = 0$, $F^{P} < 0$;

3) Plastic regime, $F^{P} = \dot{F}^{P} = 0$, $F^{D} < 0$;

4) The coupled damage-plasticity regime, $F^{D} = \dot{F}^{D} = 0$, $F^{P} = \dot{F}^{P} = 0$.

The consistency conditions can be used for particular regimes in order to express λ^{P} and λ^{D} in terms of stress or strain rates. The consistency condition for the plastic flow is adopted as:

$$\dot{F}^{P} = \frac{\partial F^{P}}{\partial \boldsymbol{\sigma}} : \dot{\boldsymbol{\sigma}} + \frac{\partial F^{P}}{\partial D} : \dot{D} = 0. \tag{20}$$

Substitute Eqs. (11a), (13), (19) into Eq. (20), the following equation can be obtained:

$$-\lambda_{P}\left(\frac{\partial F^{P}}{\partial \boldsymbol{\sigma}} : E : \frac{\partial F^{P}}{\partial \boldsymbol{\sigma}}\right) + \lambda_{D}\left(\frac{\partial F^{P}}{\partial \boldsymbol{\sigma}} : \frac{\partial^{2} \boldsymbol{\Phi}}{\partial D \partial \boldsymbol{\varepsilon}} : \frac{\partial F^{D}}{\partial Y} + \frac{\partial F^{P}}{\partial D} : \frac{\partial F^{D}}{\partial Y}\right) + \frac{\partial F^{P}}{\partial \boldsymbol{\sigma}} : E : \dot{\boldsymbol{\varepsilon}} = 0. \tag{21}$$

By an analogous procedure, the consistency condition for damage can be shown as that:

$$-\lambda_{P}\left(\frac{\partial F^{D}}{\partial Y} : : \frac{\partial^{2} \boldsymbol{\Phi}}{\partial D \partial \boldsymbol{\varepsilon}^{P}} : \frac{\partial F^{P}}{\partial \boldsymbol{\sigma}}\right) + \lambda_{D}\left(\frac{\partial F^{D}}{\partial Y} : \frac{\partial^{2} \boldsymbol{\Phi}}{\partial D^{2}} : \frac{\partial F^{D}}{\partial Y}\right) + \frac{\partial F^{D}}{\partial Y} : \frac{\partial^{2} \boldsymbol{\Phi}}{\partial D \partial \boldsymbol{\varepsilon}} : \dot{\boldsymbol{\varepsilon}} = 0. \tag{22}$$

Finally it is obtained the nonlinear incremental stress-strain relationship:

$$\dot{\boldsymbol{\sigma}} = E : \dot{\boldsymbol{\varepsilon}} - \lambda^{P} E : \frac{\partial F^{P}}{\partial \boldsymbol{\sigma}} + \lambda_{D} \frac{\partial^{2} \boldsymbol{\Phi}}{\partial D \partial \boldsymbol{\varepsilon}} : \frac{\partial F^{D}}{\partial Y}. \tag{23}$$

The last term on the right hand side of Eq. (23) represents the influence caused by inelastic damage strain $\boldsymbol{\varepsilon}^{D}$.

1.3 Discussion on the calculation of damage strain $\boldsymbol{\varepsilon}^{D}$ and dilatancy

In the theory of plasticity, the inelastic volume change, or say dilatancy, is zero. However, dilatancy is very popular in geo-engineering. In fact, dilatancy is caused by generating and propagating of cracks.

According to Rice's theory of internal variables[13], the dilatancy caused by damage tensor D is

$$d\boldsymbol{\varepsilon}_{ij}^{D} = \frac{\partial Y_{ij}}{\partial \boldsymbol{\sigma}_{kl}} dD_{kl}. \tag{24}$$

Here we call $\boldsymbol{\varepsilon}^{D}$ the damage strain. The inelastic volume strain caused by $\boldsymbol{\varepsilon}^{D}$ is the dilatancy:

$$d\boldsymbol{\varepsilon}_{v}^{d} = d\boldsymbol{\varepsilon}_{ii}^{d} = \mathrm{tr}(d\boldsymbol{\varepsilon}_{ij}^{d}). \tag{25}$$

Damage strain $\boldsymbol{\varepsilon}^{D}$ doesn't result in energy dissipation, because related energy dissipation is accounted for in the calculation by damage and its conjugate force. It should be noted that damage strain $\boldsymbol{\varepsilon}^{D}$ is also different from plastic strain $\boldsymbol{\varepsilon}^{p}$ which is usually connected with dislocation or frictional sliding.

As the dilatancy is not considered in plastic calculation, the plastic potential can be then the same as plastic yielding function.

2 Specific Coupled Elastoplasticity and Damage Model

In this section, firstly it is introduced the model of orthotropic damage. Secondly, the coupled damage-plasticity model is established with the method of homogenization.

2.1 Specific damage model

The damage model adopted here is a second-order tensor, which can describe the orthotropical damage behavior existing in geomaterial. The definition of the second-order damage tensor is given follows:

$$\mathrm{D} = \sum_{i=1}^{3} \omega_i \mathrm{n}_i \otimes \mathrm{n}_i , \tag{26}$$

where n_i is the principal direction and ω_i the principal damage value of the second-order damage tensor. A symmetric second-order damage tensor implies orthotropy with axis rotating with the principal basis of the damage tensor.

In order to consider the effect of crack closure under compression, the concept of active damage tensor[2],[14] is adopted here:

$$\left.\begin{aligned} \widetilde{\mathrm{D}} &= \mathrm{P}^{+}(\boldsymbol{\varepsilon}) : \mathrm{D}, \quad P_{ijkl} = Q^{+}_{ik} Q^{+}_{jl}, \\ \mathrm{Q}^{+} &= \sum_{i=1}^{3} h(\boldsymbol{\varepsilon}_i)\, \mathrm{p}_i \otimes \mathrm{p}_i, \quad \boldsymbol{\varepsilon}^{+} = \sum_{i=1}^{3} \boldsymbol{\varepsilon}_i \mathrm{p}_i \otimes \mathrm{p}_i, \end{aligned}\right\} \tag{27}$$

where P^{+} is the projection operator; Q^{+} is the positive definite spectral tensor; $h(\boldsymbol{\varepsilon}_i)$ is the Heaviside function; p_i is the ith principal vector of the strain tensor $\boldsymbol{\varepsilon}$.

On the basis of energy equivalent principle[1], the damaged elasticity tensor can be expressed as:

$$\mathrm{E} = (\mathrm{I} - \widetilde{\mathrm{D}}) \cdot \mathrm{E}^{0} \cdot (\mathrm{I} - \widetilde{\mathrm{D}}). \tag{28}$$

The damage potential function used here can be expressed as:

$$F^{\mathrm{D}} = \sqrt{\mathrm{Y} : \mathrm{P}^{+} : \mathrm{Y}} - \Phi_0 \leq 0, \tag{29}$$

where Φ_0 is the threshold value for F^{D}, at which damage begin to propagate.

2.2 Coupled damage-plasticity model: homogenization of Mohr-Coulomb criterion

Homogenization[12] is an effective tool to establish macroscopic relationship on the basis of microscopic analysis. Consider the representative plane element consisting of two cracks shown in Fig. 1. Plane-strain stress state is assumed.

Under the action of compressive stress σ_1 and σ_3, for the sake of general case, one crack is open, represented by Ω_n^{ad} and one is closed, represented by Ω_n^{id}. Here the superscript ' ad ' means active damage, ' id ' means inactive damage. n is the normal direction vector and t is the tangential direction vector.

At the microscope, for the intact material consisted in this element, the Mohr-Coulomb criterion is

$$F_t^{\mathrm{P}} = \tau_n^{\mathrm{t}} - \mu_t \sigma_n^{\mathrm{t}} - c^{\mathrm{t}} = 0, \tag{30}$$

where τ_n is the shear stress and σ_n is the normal stress acting on the sliding plane; c is shear strength; μ is the friction coefficient of this material. Subscript t and superscript t denote intact material.

For the inactive damage section, or say the closed crack, the Mohr-Coulomb criterion is

$$F_{\mathrm{id}}^{\mathrm{P}} = \tau_n^{\mathrm{id}} - \mu_{\mathrm{id}} \sigma_n^{\mathrm{id}} - c^{\mathrm{id}} = 0. \tag{31}$$

For the active damage section, or say open crack, the Mohr-Coulomb criterion is

$$F_{\mathrm{ad}}^{\mathrm{P}} = \tau_n^{\mathrm{ad}} - \mu_{\mathrm{ad}} \sigma_n^{\mathrm{ad}} - c^{\mathrm{ad}} = 0. \tag{32}$$

In the framework of homogenization theory, the following equivalent relationship can be established between the macroscopic behavior and analysis of microstructure:

$$\tau_n = \tau^t_n(1 - \Omega^{ad}_n - \Omega^{id}_n) + \tau^{id}_n\Omega^{id}_n, \quad (33)$$

$$\sigma_n = \sigma^t_n(1 - \Omega^{ad}_n - \Omega^{id}_n) + \sigma^{id}_n\Omega^{id}_n, \quad (34)$$

where $\Omega^{ad}_n = \tilde{D}_{ij}n_in_j$, $\Omega^{id}_n = (D_{ij} - \tilde{D}_{ij})n_in_j$ are the components of inactive damage tensor and active damage tensor respectively.

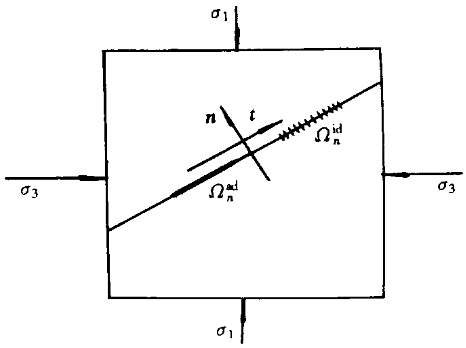

Fig. 1 The 2-D representative element

It is assumed that the stress acting on the closed crack surface is 0 and the stress acting on the closed crack surface is the same as that acting on the intact material, i.e.:

$$\sigma^{ad}_n = 0, \quad \sigma^t_n = \sigma^{id}_n = \sigma_n(1 - \Omega^{ad}_n)^{-1}, \quad (35)$$

where σ_n is the normal stress acting on the sliding plane at macroscale.

The homogenized Mohr-Coulomb criterion can be written as

$$\tau_n = c^* + \mu^*\sigma_n, \quad (36)$$

where

$$\mu^* = (1 - \Omega^{ad}_n)[\mu^t(1 - \Omega^{ad}_n - \Omega^{id}_n) + \mu^{id}\Omega^{id}_n], \quad (37)$$

$$c^* = c^t(1 - \Omega^{ad}_n - \Omega^{id}_n) + c^{id}\Omega^{id}_n. \quad (38)$$

Owing to the orthotropic behaviour caused by damage, it is necessary to present the Mohr-Coulomb criteria for all the critical planes on where plastic flow, or say frictional sliding, occurs. The determination of the direction of critical plane will be presented in another paper.

Denotes the direction of the ith critical plane with normal directional vector v_i, $i = 1, N$, where N is the total number of critical planes. Then Eq. (37) can be written as

$$\tau_{ni} = c^*_i + \mu^*_i\sigma_{ni}, \quad (39)$$

where the subscript i means the component value in the ith critical plane of the above mentioned tensors:

The orthotropic plastic evolution equation for 3-D problem can be written as follows:

$$\dot{\varepsilon}^P = \sum_{i=1}^{3}\lambda^P_i\frac{\partial F^P_i}{\partial\sigma}, \quad (40)$$

where λ^P_i can be solved from Eq. (21) and Eq. (22) for a given $\dot{\varepsilon}$ with $F^P = F^P_i$.

Together with the equations from Eq. (26) to Eq. (40), the specific coupled model is formed.

3 Conclusions

A phenomenological orthotropic damage model coupled with plasticity has been presented for geomaterials. The characteristics of this study are

1) The theory for orthotropic damage model coupled with plasticity has been formulated on the basis of irreversible thermodynamics. Associated plastic flow rule is adopted for the fricitional sliding of plasticity and damage evolution while the dilatancy effect is calculated separately by damage strain.

2) The damage tensor has been introduced into the Mohr-Coulomb criterion on the basis of

the analysis of homogenization theory. Both the degradation of shear strength and that of friction angle have been taken into consideration in this model.

3) The damage strain is adopted to account for the inelastic strain related to damage tensor, and further, dilatancy is calculated through damage strain. Therefore, dependent flow rule can be used in the plastic calculation.

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