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Discussion

An interfacial mechanical model for the analysis of earthquake*

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Abstract

In this paper, the shear beam model for analysis of interface failure under joint action of anti-plane shearing and lateral compression and its principal behavior were briefly introduced. The calculation of energy release that is related to the strength of earthquake was presented by using the shear beam model. The sudden increase of load resulted from the ″stress locking″ at the interface layer in the reloading process was investigated. At the end of the paper, discussions on the mechanism of earthquake were given out.

Key words: interface failure; shear beam model; stress locking; sudden variation of load; instability; earthquake
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Introduction

China is a country suffered frequent earthquake disasters. The investigation on the mechanism of earthquake is of great significance. In the recent decades numerous researchers had carried out a great deal of research works on the theory of seismology and related *in-situ* observations (DU, SHAO, 1999; JIANG, *et al*, 1998; JIAO, *et al*, 1999; ZENG, SONG, 1998). However the mechanism of earthquake and relevant calculations still remain unsolved and problems still exists in prediction of earthquake.

It is a common knowledge that earthquake is resulted from the unstable relative sliding of the rock mass in the two sides of a macro fault. It is seen from macro scope that the earthquake is not related to the fracture of intact rock layer because earthquake always happens along the same fault in the same region. Therefore it can be concluded that earthquake is a kind of structural unstable behavior of rock layer caused by shear failure under the joint action of shearing and lateral compression. The investigation on the mechanism of the shear failure of the geo-interface under this kind of joint action and on the evolution of related mechanical variables are of important significance to seismic research.

A detailed survey of research on the topics of interface crack can be found, for instance, in the

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article by Hutchinson and Suo (1991) who discussed mixed-mode crack propagation using the Griffith energy condition. But the study on the topics of shear failure of interface crack under the joint action of shear and lateral compression is rare. Recently Mroz and SHEN (1999) applied the cohesive crack model to the interface crack and presented their shear beam model for the analysis of interface failure under the action of anti-plane shear and lateral compression. The results presented by Mroz and SHEN (1999) and SHEN and Mroz (2000) indicated that the shear beam model has the following merits:① The ″stress locking″ phenomenon on the interface can be re-produced using this model; ② The load-displacement behavior of the structure on the equilibrium path can be analytically calculated using this model. This model primarily offered an effective vehicle for the analysis of the evolution of mechanical variables on the interface in the complex loading process.

In this paper the shear beam model (Mroz, SHEN, 1999) is briefly introduced firstly. By means of the shear beam model the sudden increase of load due to the stress-locking phenomenon at the interface of rock layer is presented, and the calculation of energy release of the structure during the stage of unstable deformation is given out. Finally efforts are made to connect the above theory of mechanics with the phenomenon of earthquake and suggestions are presented.

1 Anti-plane shear and shear beam model

The double-shear-plates of length L, width b, and thickness t, bonded by cohesive layer of thickness $2h$, $h<<t$, is shown in Figure 1. The uniform compressive traction $s_{zz}=s_n$ is assumed to act at the surface of upper plate. The plate is loaded by the anti-plane shear stress t (0, z), ∂t (0, z)/∂z = const at the end section x=0. The other end section at x=L remains traction free. The initiation of damage process zone and crack will start from the end x=0, and propagate toward the other end x=L. Assuming the transverse dimension b is to be much larger than the plate thickness t, the flexural effect can be neglected and the state of anti-plane shear can be assumed that there are two non-vanishing shear stress component t_{yx} and t_{yz}, so that the equilibrium equation is

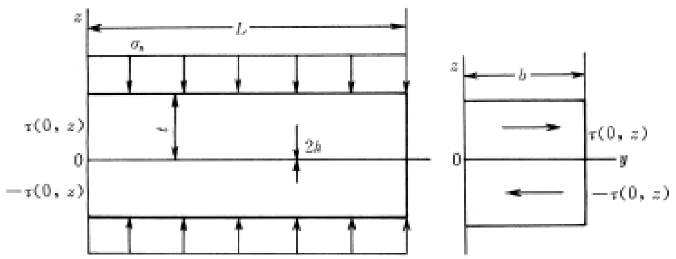

Figure 1 Double-shear-plates bonded by cohesive layer

$$\frac{\partial t_{yx}}{\partial x}+\frac{\partial t_{yz}}{\partial z}=0 \tag{1}$$

and $s_{zz}=s_n$=const is the initial stress induced by the lateral compressive traction along the z-axis. Denoting by $w=w(x, z)$, the displacement field along the y-axis can be written by using the Hooke′s law:

$$t_{yz}=-G_2\frac{\partial w}{\partial z} \qquad t_{yx}=-G_1\frac{\partial w}{\partial x} \tag{2}$$

Substituting equation (2) to equation (1), the equilibrium equation (1) takes the form

$$G_1\frac{\partial^2 w}{\partial x^2}+G_2\frac{\partial^2 w}{\partial z^2}=0 \tag{3}$$

where G_1, G_2 are the shear moduli along x- and z- axes. Due to the anti-symmetry of the problem relative to the central line of the interface layer, we need consider only the upper part of plate. The boundary conditions at the interface z =0: $\tau_{yz}=-t_f$, where t_f is the interface shear stress at z =0 at the upper boundary $z = t$, t_{yz}=0, and at the transverse boundaries x=0, $t_{xy}(0)=f_0(y, z)$; $x=L$, $t_{xy}(L)$=0 should be satisfied.

The shear beam model presented by Mroz and Shen (1999) is shown in Figure 2. A beam bonded to a rigid foundation. Between the beam and the rigid foundation, there is a cohesive layer of thickness h. It is loaded by the shear force $T=t_{yx}(0)A$, $A=bt$, at the end section x=0. The other end section at $x=L$ remains traction free. The flexural effect in the beam is neglected. In z-direction the shear displacement is assumed to be constant, that is, $w=w(x)$ and $\partial w/\partial z$ =0, $G_2\to\infty$, $G_1=G$ and the beam is an elastic beam, *i.e.*,

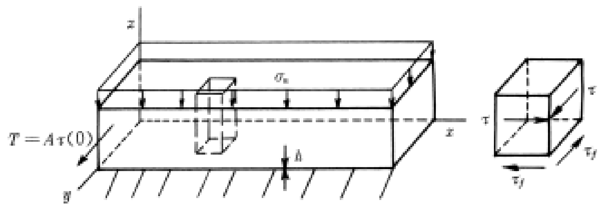

Figure 2 Shear beam model

$$t=-G\frac{\mathrm{d}w}{\mathrm{d}x} \tag{4}$$

The shear stress t of the elastic beam and shear stress t_f of cohesive layer form a self-balanced system as that described in Jarzebowski and Mroz (1994). Writing the equilibrium equation for the elastic beam interacting with the interface cohesive layer as

$$A\frac{\mathrm{d}t}{\mathrm{d}x}+bt_f=0 \tag{5}$$

where $A=bt$ denotes the transverse cross section area. Substitute equation (4) into equation (5) and introduce the non-dimensional operation on length variables referred to the plate thickness t, the regularized equilibrium equation takes the form

$$\frac{\mathrm{d}t}{\mathrm{d}x}+t_f=0 \qquad \frac{\mathrm{d}^2 w}{\mathrm{d}x_2}-\frac{t_f}{G}=0 \tag{6}$$

It was proved by Mroz and Shen (1999) that the shear beam model represented by equation (6) is equivalent to the simplified anti-plane shear problems under uniform lateral compression represented by equation (1) to (3).

2 Calculation of energy release

The behavior of the shear beam, as the damage process zone reaches the free end is defined by Mroz and Shen (1999) as ″end-zone behavior″. At the ″end-zone″ stage, the shear beam undergoes unstable deformation process related to the material properties of the interface material. There are two different models of instability can be distinguished, *i.e.*, snap-back and snap-through. The snap-back or snap-through response depends on the sign of the discriminant equation that was derived rigorously by Mroz and Shen (1999). The discriminant equation consists of parameters of

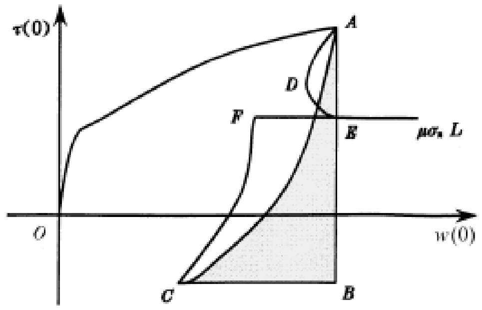

Figure 3 Schematic diagrams for snap-back behavior

material property, the geometrical parameters and the parameters of external environment. As the condition for snap-back is satisfied, the calculation of energy release comes to us because the reasonable calculation of energy release is the premise to the application of a mechanical model to seismic analysis. Based on the results (Mroz, Shen, 1999), Figure 3 shows two groups of schematic diagrams of snap-back behavior. The schematic diagram *ADE* is the case for snap-back and it was reported in numerous references, such as Yang and Ravi-Chander (1998). But under some loading condition and the given parameters, the results mathematically obtained with shear beam model can be shown by diagram *ACFE* in which the lowest load, point *C*, is below the final value $m s_n L$.

Yin (1993) and Hua (1989) presented their theories for the calculation of energy release in the process of unstable deformation of geo-structure respectively. In this paper it is believed that the calculation of energy release of unstable deformation process, such as earthquake, being carried out merely using the properties of constitutive relationship is not perfect. This is because earthquake, together with the other unstable deformation, is a kind of synthetical structural behavior. The strength of energy release, or the intensity of the earthquake, depends not only on material parameters, but also on the geometrical parameter and the value of crustal stresses. Furthermore, it is believed in this paper that the process of snap-back is the process in which the structure release its energy stored in previous deformation process to the external environment. The mark of the energy release is the opposite displacement at the loading end of the structure, and the amount of the released energy E^R is represented by the shadow area of *ABCA* in Figure 3. Its calculation equation is

$$E^{R} = \int_{w(0)_1}^{w(0)_2} t(0)\mathrm{d}w(0) \tag{7}$$

where $w(0)_1$ is the displacement value at the loading end when the opposite displacement starts to appear, $w(0)_2$ is the value at the loading end when the opposite displacement disappears; $t(0)$ is the shear stress at the loading end and is a function of variables including crustal stress s_n, material parameters C_0 and geometrical parameters t, L, *etc.* By using E^R, the intensity of earthquake can be obtained. Therefore, equation (7) is physically reasonable for the calculation of the amount of released energy.

3 Analysis on the mechanism of sudden increase of load

In many cases the failure of structure is caused by impacting load. For rock mass, impact load can be caused by explosion, but more popularly in practice, it is related to the famous phenomenon of stress locking that resulted from cyclic geological movement.

Figure 4 shows the distribution of t_f along the interface layer and the positions of separating points between different regions of t_f after a cycle of loading–unloading–reloading process. In Figure 4, s_{51} represents the tail point of loading damage zone ahead of macro interface crack, s_{52} represents the frontier of loading damage zone; s_{31} represents the frontier of the unloading reverse sliding zone and s_{32} represents the frontier of the second-damage zone. The so-called second-damage zone is the layer zone that damaged twice during loading process and unloading process; s_{51} represents the tail point of the reloading third-damage zone and s_{52} represents the frontier of the reloading process zone. The so-called third-damage zone is the layer zone that damaged three times during the loading-unloading-reloading process. In the unloading process, owing to the reaction of the elastic beam, reverse frictional sliding on the interface layer starts from the loading end x=0 and proceeds towards the free end. For the shear beam model, the self-balanced relationship holds between the shear stress t in the elastic beam and t_f on the cohesive interface layer. From equation (6) the value of shear stress t at the end point x=0 is the integral of t_f along the entire length of the interface layer and there is

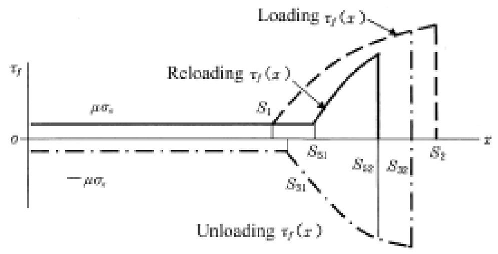

Figure 4 Positions of separating points between different regions of t_f

$$t(0) = \int_0^L t_f \mathrm{d}x \tag{8}$$

The increment of the load in the reloading process is

$$\Delta t(0) = \int_0^{s_{52}} \Delta t_f \mathrm{d}x \tag{9}$$

The form of derivative of equation (9) with respective to time is

$$\frac{\mathrm{d}[t(0)]}{\mathrm{d}t} = \frac{\partial[\Delta t(0)]}{\partial s_{52}}\frac{\mathrm{d}s_{52}}{} \cdot \frac{\mathrm{d}s_{52}}{\mathrm{d}t} \tag{10}$$

It can be proved that the value of variable s_{52} is finite. But its time rate $\mathrm{d}s_{52}/\mathrm{d}t$ is infinite as s_{52} reaches s_{32}, which is the tail point of ″stress locking area″. This can be illustrated with the help of Figure 4.

In Figure 4, the frontier s_{52} of the reloading third-damage zone proceeds towards the end x=L. As s_{52} reaches s_{32}, the ″locked″ stress is connected with the reloading third-damage zone and results in the jump of s_{52} from one side of ″stress locking area″ to its other side. In this process there is

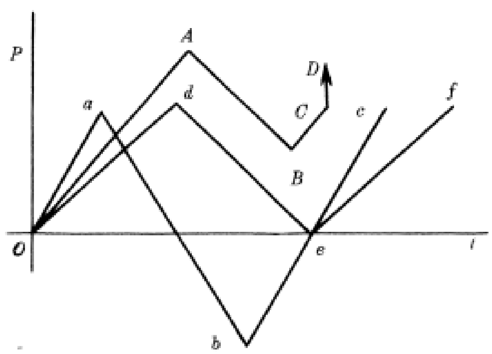

Figure 5 Illustration of cyclic loading (shear force P *versus* time t)

$$\frac{\mathrm{d}s_{52}}{\mathrm{d}t} \to \infty \tag{11}$$

Together with equation (11), it can be obtained

$$\frac{\mathrm{d}[\Delta t(0)]}{\mathrm{d}t} \to \infty \tag{12}$$

this jump of load increment is shown in Figure 5, the time-load diagram, as the vertical line CD.

4 Conclusions

The shear beam model can reproduce principal mechanical characteristics of the anti-plane shear failure of rock mass under the joint action of lateral compression and anti-plane shear. It can be applied to the investigation of mechanism of earthquake and evolution of related variables. This paper has discussed numerous problems related to analysis of earthquake using shear beam model and the following conclusions are obtained:

(1) Earthquake is a kind of structural behavior. Its intensity should be determined on the basis of the synthetical influence of the parameters including crustal stress, geometrical parameter of local structure and material properties. The calculation equation of energy release given in this paper has accounted for all the factors mentioned above and is physically more reasonable.

(2) When the frontier of the reloading third-damage zone approaches the area of ″stress locking″, sudden load will occur and it is indicated that consequent earthquake is possible. Therefore observation on the variation of the shear stress t_f on the interface layer, especially the movement of the frontiers of various shear stress regions, is of important significance for the prediction of impending earthquake.

(3) The sudden load increment may occur several times if the interface layer consists of plural ″stress locking area″. This case can be true as long as the interface is long enough. This may be the sources of energy for plural aftershocks. Consequently it can be concluded that, the accurate measurement on the distribution of the shear stress along the interface of the fault can result in accurate prediction on the intensity of earthquake and the number of aftershocks.

(4) The *in-situ* seismic survey should concern more on the distribution of shear stress on the interface of fault. Although the mechanical message recorded by the intact rock mass is related to earthquake, the message recorded by the interface layer of fault is more important because the earthquake is resulted from the instability behavior of interface of rock mass.

(5) When the sudden increment of load occurs, the frictional sliding on the interface will be accelerated. The frictional sliding will result in energy dissipation and cause increment of local temperature; if this area is connected with external water sources, the temperature of water will become higher. If the frictional sliding results in the wear and tear of the rough interface, the acoustic emission phenomenon, which usually is a warning sign of a earthquake, will occur.

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