A Plasticity-based Damage Model for Concrete

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Abstract: A plasticity-based damage model is proposed for inelastic analysis of structural concrete in this paper. Drucker-Prager-type plasticity is adopted in the formulation of the constitutive equations. Numerical validations have been performed for the proposed model with a driver subroutine developed in this study. Numerically predicted behaviour with the proposed model seems reasonably accurate for the cases of uniaxial tension and uniaxial compression when compared with experimental data in the literature. Numerical tests for multi-axial compression cases under various states of hydrostatic confinement stress have been carried out. Results indicate the validity of the proposed model for simulating plastic damage phenomena under various loading paths.

Key words: damage, plasticity, constitutive model, numerical analysis, concrete.

1. INTRODUCTION

The aim of this investigation was to build a constitutive model that can simulate the history-dependent plastic damage behaviour of massive structural concrete under confinement. There are several isothermal damage models for concrete described in the existing references. A brief review of some typical damage models is as follows:

One of the popular models used in practice is the Mazars-PijaudierCabot damage model (Mazars and Pijaudier-Cabot, 1989). However, because damage evolution in this model is expressed in a holonomic manner with respect to strain loading, no incremental form has been presented for a damage evolution law. The damage is thus uncoupled from the evolution of plastic strain. Consequently this model can not be used for simulating history-dependent plastic damage problems.

Another popular model is the damage model by de Borst et al. (1999) in which its damage evolution law is based on total quantity of equivalent strain. In practice, there is plenty of strength data available in terms of equivalent stress and/or fracture energy, but not so much existing data for strain-based criteria. Thus, the equivalent-strain-based damage model will require special tests. Consequently, for this reason, a plasticity-based damage model with an equivalent stress based loading criterion is more practical for engineering purposes than the model by de Borst et al. (1999).

One of the plasticity-based damage models is the so-called Bacelona model which was reported by Lubliner et al. (1989), and recently adopted by Lee and Fenves (1998). In that model, a holonomic relationship between damage and equivalent plastic strain is proposed. Two damage variables are adopted for tensile damage and compressive damage respectively. However, damage evolution is also uncoupled from evolution of plastic strain in this model.

Anisotropic damage models of vector form and of 2nd order tensor form have also been investigated by a few researchers (see Swoboda et al. 1998; Shen et al. 2001). However, because of the difficulties in the simulation of

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the experimental phenomena with concrete specimens, together with its in-convenient on computational aspects, more research work is needed for these models to be adopted.

The model adopted for this paper is constructed on the basis of the plasticity-based damage model reported by Saanouni et al. (1994). Although this model was proposed for the simulation of plastic damage phenomena in metals, it is still an attractive model for the purpose of this study because of the following advantages over the other available damage models: firstly, in this model, the damage evolution is closely connected to the increase of plastic strain, but is also influenced explicitly by the elastic strain; secondly, damage evolution is coupled with increase of plastic strain; finally, this model is relatively easy to modify to make it suitable for the simulation of plastic damage phenomena in concrete, which is quite different from that in metals. The generalized Drucker-Prager criterion, introduced in Menétrey and Willam (1995) for plastic loading, together with its plastic potential for non-associated plastic flow rule, is referred here.

In section 2 of this paper, the equations of the proposed constitutive model are given, and a driver subroutine for validation of the constitutive model is developed. In section 3, the results of numerical tests of the proposed model are given for some typical loading cases. Some remarks are given in section 4.

2. FORMULATION OF THE PROPOSED MODEL

2.1. Fundamental Equations of the Plasticity-based Damage Model

With the '*Energy Equivalence Principle*', the fundamental relationships of the plasticity-based damage model proposed by Saanouni et al (1994) are listed in the following Eqn 1 as:

$$\begin{cases} \tilde{\sigma}_{ij} = \dfrac{\sigma_{ij}}{1-D}, \quad \tilde{\varepsilon}_{ij}^{e} = (1-D)\varepsilon_{ij}^{e}, \quad \tilde{E}_{ijkl}^{0} = \dfrac{E_{ijkl}}{(1-D)^2} \\ Y = (1-D)E_{ijkl}^{0}\varepsilon_{ij}^{e}\varepsilon_{kl}^{e}, \quad \dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\dfrac{\partial \tilde{Q}}{\partial \sigma_{ij}}, \quad \dot{D} = \dot{\lambda}\dfrac{\partial F}{\partial Y} \end{cases} \tag{1}$$

where σ_{ij} and ε_{ij} are total stress and strain tensors respectively, superscript p stands for plastic and e for elastic quantities, the overhead tilt ~ represents the quantities for fictitious net materials, λ is the inelastic multiplier, D is a damage variable, Y is the damage conjugate force, E_{ijkl}^{0} is the elasticity tensor of the intact material, F is a plastic damage potential function, $\tilde{Q}$ is the plastic part of the potential in the effective stress space.

It can be seen in the above equations that the damage evolution is accompanied by plastic strain increase, and its size of increment is dependant on its elastic strain tensor via the damage conjugate force.

2.2. Specification of the Constitutive Model for Drucker-Prager Model Coupled with Damage

With reference to the generalized Drucker - Prager criterion introduced in Menétrey and Willam (1995), the plastic damage loading condition is defined in the effective stress space in the following form:

$$\tilde{f} = \alpha_F \tilde{I}_1 + \tilde{J}_2^{1/2} - \left[k + k_\infty\left(1 - e^{-b\lambda}\right)\right] = 0 \le 0 \tag{2}$$

where $\tilde{I}_1$ is the sum of the effective principal stresses, $\tilde{J}_2$ is the second invariant of the deviatoric effective stress tensor, k is the initial shear strength constant, k_∞ is the strain hardening limit of the fictitious net material, which corresponds to infinite equivalent plastic strain, i.e. $\lambda \to \infty$, and α_F is a material constant designed for pressure-sensitive properties, and parameter b is a model constant which can be determined by fitting experimental phenomena.

$\tilde{Q}$, the plastic part of the potential in the effective stress space, is given by:

$$\tilde{Q} = \alpha_Q \tilde{I}_1 + \tilde{J}_2^{1/2} - \left[k + k_\infty\left(1 - e^{-b\lambda}\right)\right] \tag{3}$$

where α_Q is the dilatancy constant for the non-associated flow rule if $\alpha_Q \ne \alpha_F$.

Here the following form of plastic damage potential function F is adopted in order to have non-associate plastic flow in the effective stress space (Saanouni et al, 1994):

$$F = \tilde{Q} + \frac{S}{s+1}\left(\frac{Y}{S}\right)^{s+1}(1-D)^{\phi} \tag{4}$$

where s, S, ϕ are material parameters, Y is the damage conjugate force. Consequently plastic strain increments are obtained as:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\frac{\partial F}{\partial \sigma_{ij}} = \dot{\lambda}\frac{\partial \tilde{Q}}{\partial \sigma_{ij}} \tag{5}$$

where

$$\frac{\partial \tilde{Q}}{\partial \sigma_{ij}} = \frac{1}{(1-D)}\left(\alpha_Q \delta_{ij} + \frac{s_{ij}}{2\sqrt{J_2}}\right) \tag{6}$$

with $\tilde{s}_{ij}$ as the deviatoric stress tensor. The damage evolution law can be derived as:

$$\dot{D} = \dot{\lambda}\left(\frac{Y}{S}\right)^{s}(1-D)^{\phi} = \dot{\lambda}\overline{Y} \tag{7}$$

where

$$\overline{Y} = \left[\frac{(1-D)E_{ijkl}^{0}\varepsilon_{ij}^{e}\varepsilon_{kl}^{e}}{S}\right]^{s}(1-D)^{\phi} \tag{8}$$

The parameters used in this model are: $E, \nu, k, k_\infty, b, \alpha_F, \alpha_Q$ for elasto-plasticity, and s, S, ϕ for damage.

2.2. Constitutive Behaviour for a Finite Strain Increment $\Delta\varepsilon_{ij}$

With the above constitutive model, the constitutive behaviour can be derived for a known initial stress state $(\sigma_{ij}, \varepsilon^p_{ij}, D)$ and a given strain increment $\Delta\varepsilon_{ij}$. The stress increment can be obtained by undertaking a total differential operation over the total stress tensor in the above Eqn 1, together with a subsequent linearization over the time increment Δt, thus

$$\begin{aligned}\Delta\sigma_{ij} &= E^0_{ijkl}(1-D)^2\left(\Delta\varepsilon_{kl} - \Delta\varepsilon^p_{kl}\right) \\ &\quad - 2E^0_{ijkl}(1-D)\left(\varepsilon_{kl} - \varepsilon^p_{kl}\right)\Delta D \\ &= -\Delta\lambda\left[E^0_{ijkl}(1-D)^2\frac{\partial\tilde{Q}}{\partial\sigma_{kl}} + 2E^0_{ijkl}(1-D)\varepsilon^e_{kl}\overline{Y}\right] \\ &\quad + E^0_{ijkl}(1-D)^2\Delta\varepsilon_{kl}\end{aligned} \tag{9}$$

The plastic damage multiplier $\dot{\lambda}$ can be determined explicitly using a consistency condition. Its expression is:

$$d\lambda = \frac{\dfrac{\partial\tilde{f}}{\partial\sigma_{ij}}E^0_{ijkl}(1-D)^2}{\left[\dfrac{\partial\tilde{f}}{\partial\sigma_{ij}}E^0_{ijkl}(1-D)^2\dfrac{\partial\tilde{Q}}{\partial\sigma_{kl}} + 2\dfrac{\partial\tilde{f}}{\partial\sigma_{ij}}E^0_{ijkl}(1-D)\varepsilon^e_{kl}\overline{Y} - \dfrac{\partial\tilde{f}}{\partial D}\overline{Y} - \dfrac{\partial\tilde{f}}{\partial\lambda}\right]}d\varepsilon_{kl} \tag{10}$$

For a given strain increment $\Delta\varepsilon_{ij}$, the stress tensor increment can be obtained as:

$$\begin{aligned}\Delta\sigma_{ij} &= -\frac{A_{kl}}{B}\left[E^0_{ijrs}(1-D)^2\frac{\partial\tilde{Q}}{\partial\sigma_{rs}} + 2E^0_{ijrs}(1-D)\varepsilon^e_{rs}\overline{Y}\right] \\ &\quad \times\Delta\varepsilon_{kl} + E^0_{ijkl}(1-D)^2\Delta\varepsilon_{kl}\end{aligned} \tag{11}$$

where

$$\begin{cases} A_{kl} = \dfrac{\partial\tilde{f}}{\partial\sigma_{ij}}E^0_{ijkl}(1-D)^2 \\ B = \dfrac{\partial\tilde{f}}{\partial\sigma_{ij}}E^0_{ijkl}(1-D)^2\dfrac{\partial\tilde{Q}}{\partial\sigma_{kl}} \\ \qquad + 2\dfrac{\partial\tilde{f}}{\partial\sigma_{ij}}E^0_{ijkl}(1-D)\varepsilon^e_{kl}\overline{Y} - \dfrac{\partial\tilde{f}}{\partial D}\overline{Y} - \dfrac{\partial\tilde{f}}{\partial\lambda} \end{cases} \tag{12}$$

The algorithmic tangential stiffness tensor can be deduced as:

$$\begin{aligned}E^{ep}_{ijkl} &= \frac{\partial\Delta\sigma_{ij}}{\partial\Delta\varepsilon_{kl}} = E^0_{ijkl}(1-D)^2 \\ &\quad - \left[E^0_{ijrs}(1-D)^2\frac{\partial\tilde{Q}}{\partial\sigma_{rs}} + 2E^0_{ijrs}(1-D)\varepsilon^e_{rs}\overline{Y}\right]\frac{A_{kl}}{B}\end{aligned} \tag{13}$$

The elasto-plastic damage loading condition for a given strain increment $\Delta\varepsilon_{ij}$ can be expressed conceptually in the effective stress space as:

$$\tilde{f} = \tilde{f}^0 + \frac{\partial\tilde{f}}{\partial(\Delta\lambda)}\cdot\Delta\lambda \le 0 \tag{14}$$

where $\tilde{f}^0$ is the value of the yielding function at the starting effective stress state $\tilde{\sigma}^0_{ij}$. Using the Eqn 2, the following relationship can be obtained:

$$\frac{\partial\tilde{f}}{\partial(\Delta\lambda)} = \frac{\partial\tilde{f}}{\partial\sigma_{ij}}\frac{\partial\sigma_{ij}}{\partial(\Delta\lambda)} + \frac{\left(\alpha I_1 + \sqrt{J_2}\right)}{(1-D)^2}\frac{\partial D}{\partial(\Delta\lambda)} + \frac{\partial\tilde{f}}{\partial\lambda}\frac{\partial\lambda}{\partial(\Delta\lambda)} \tag{15}$$

With Eqn 2 and the fundamental relationships in Eqn 1, the tensors and vectors on the right hand side of Eqn 15 are obtained as

$$\frac{\partial\tilde{f}}{\partial\sigma_{ij}} = \frac{1}{(1-D)}\left(\alpha\delta_{ij} + \frac{s_{ij}}{2\sqrt{J_2}}\right) \tag{16}$$

$$\begin{aligned}\frac{\partial\sigma_{ij}}{\partial(\Delta\lambda)} &= -\left[E^0_{ijkl}(1-D)^2\frac{\partial\tilde{Q}}{\partial\sigma_{kl}}\right. \\ &\quad \left. + 2E^0_{ijkl}(1-D)\varepsilon^e_{kl}\left(\frac{Y}{S}\right)^s(1-D)^\phi\right]\end{aligned} \tag{17}$$

$$\frac{\partial\tilde{f}}{\partial(\lambda)} = -bk_\infty e^{-b\lambda},\quad \frac{\partial D}{\partial(\Delta\lambda)} = \left(\frac{Y}{S}\right)^s(1-D)^\phi \tag{18}$$

The formulation of the Newton-Raphson iteration equation between $\Delta\lambda$ and $\tilde{f}$ is formed as:

$$\Delta\lambda = \Delta\lambda_0 - \tilde{f}_0\left(\frac{\partial\tilde{f}}{\partial\lambda}\right)^{-1} \tag{19}$$

where $\tilde{f}^0$ is the value of the yielding function at the starting effective stress state $\tilde{\sigma}_{ij}^0$.

3. NUMERICAL VALIDATION AT LOCAL LEVEL

With reference to the algorithm proposed in Hashash et al. (2002) for elasto-softening plasticity, here a driver subroutine is developed for the purpose of validation of the 3-dimensional constitutive model for plasticity coupled with damage. Its principle is that a mixed loading condition is applied with $\varepsilon_{11} = \varepsilon_{11}(t)$ and $\sigma_{22} = \sigma_{33} =$ constant, which means that an axial strain loading will be applied incrementally under a constant confinement stress in the other two directions. Strain loading is applied elastically in the 11-direction, while the self-equilibrium mechanism at this material point will result in variation of lateral strains nonlinearly in order to keep the lateral confinement constant. The details of the numerical calculations are given in the following context.

3.1. Iteration Procedure for the Constitutive Model Validation: External Equilibrium Iteration.

The function of the *external equilibrium iteration* is for a known stress and strain state and a set of internal variables $(\sigma_{ij}, \varepsilon_{ij}, \varepsilon_{ij}^p, D)$, to apply a load increment $(\Delta\varepsilon_1, \Delta\sigma_2, \Delta\sigma_3)$, and to find out quasi-elastically the response of $(\Delta\sigma_1, \Delta\varepsilon_2, \Delta\varepsilon_3, D)$ by an iterative procedure.

The following quasi-elastic equations (i.e. elastic relationship for a finite time increment Δt) are adopted in the calculation:

$$\begin{Bmatrix} \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \end{Bmatrix} = \begin{bmatrix} E_{2222} & E_{2233} \\ E_{3322} & E_{3333} \end{bmatrix}^{-1} \left[\begin{Bmatrix} \Delta\sigma_{22} \\ \Delta\sigma_{33} \end{Bmatrix} - \begin{Bmatrix} E_{2211} \\ E_{3311} \end{Bmatrix} \Delta\varepsilon_{11} \right] \tag{20}$$

where E_{ijkl} is the elasticity tensor of damaged material as expressed in Eqn 13.

UMAT is the constitutive module which checks the loading state and makes elastic and/or elasto-plastic damage calculations. The subroutine CONSTITUERE, which performs the constitutive integration and is introduced in detail in the following sub-section, is called in UMAT. The loading elasto-plastic-damage stiffness (E_{ijkl}^{epd}), which is also known as algorithmic tangential stiffness, is updated after every iteration, and is used in the quasi-elastic calculation of $\Delta\varepsilon_2$ and $\Delta\varepsilon_3$ at every first iteration step at each loading increment.

3.2. Iteration Procedure for the Constitutive Model Validation: Internal Elasto-plastic Damage Iteration

The solution steps adopted in the CONSTITUERE subroutine are:

- Step 1: Initiate the stress state and state of all the internal variables: $\sigma_{ij}^0, \varepsilon_{ij}^0, \varepsilon_{ij}^p, D_0$
- Step 2: Apply strain increment $\Delta\varepsilon_{ij}$, with $\Delta\varepsilon_{ij} = 0$ for $i \neq j$ obtained from the outer global equilibrium iteration;
- Step 3: Calculate $\Delta\lambda_0$ with given initial stress state, strain increment and linearized Eqn 10;
- Step 4: With this $\Delta\lambda_0$ obtained in step 3, calculate consequently the following quantities:

$$\varepsilon_{ij}^e = \varepsilon_{ij}^{e(0)} + \Delta\varepsilon_{ij} - \Delta\lambda_0 \frac{\partial\tilde{Q}}{\partial\sigma_{ij}^0}$$

$$\varepsilon_{ij} = \varepsilon_{ij}^{(0)} + \Delta\varepsilon_{ij}, \qquad \varepsilon_{ij}^p = \varepsilon_{ij} - \varepsilon_{ij}^e$$

$$D = D^{(0)} + \Delta\lambda_0\bar{Y},$$

$$\text{with } \bar{Y} = \left[\frac{\left(1 - D^{(0)}\right)E_{ijkl}^0\varepsilon_{ij}^e\varepsilon_{kl}^e}{S\gamma}\right]^s \left(1 - D^{(0)}\right)^\phi$$

$$\sigma_{ij} = E_{ijkl}^0(1 - D)^2\varepsilon_{ij}^e$$

- Step 5: With Eqn 15 and 19, calculate iteratively the plastic-damage multiplier with the following equations:

$$\Delta(\Delta\lambda) = -\tilde{f}_0\left(\frac{\partial\tilde{f}}{\partial\lambda}\right)^{-1}, \qquad \Delta\lambda = \Delta\lambda_0 + \Delta(\Delta\lambda)$$

- Step 6: Check convergence: if $\Delta(\Delta\lambda) \le \text{Tolerance}$, cease the iteration and carry on with the next load increment;
- Otherwise, make

$$\Delta\lambda_0 = \Delta\lambda$$

Return to step 2 to carry on with the next iterative calculation up to the maximum iteration limit.

3.3. Numerical Examples

In this section, numerical validations of the proposed constitutive model at local level are carried out with the driver subroutine for 3 kinds of typical loading cases, i.e., (1) uniaxial tension, (2) uniaxial compression, and (3) compression under confinement.

The following values of material parameters are adopted in the calculation:

E = 31140MPa, $\nu = 0.2$, $\alpha_F = \alpha_Q = 0.1$, k = 2.0MPa, s = 1, S = 10^{-10} for tension and 5.5×10^{-5} for compression, $\phi = -1.0$, b = 3 for tension and 300 for compression, k_∞ = 1MPa for tension and 90MPa for compression.

3.3.1. Uniaxial Tension

The following Figures 1 and 2 show the stress-strain behaviour and damage evolution response of the model under uniaxial tension.

It is seen in Figure 1 that the agreement of the predicted results of the model and the experimental results reported in Lee and Fenves (1998) are quite good at the pre-peak stage. However the predicted results at the post-peak stage for uniaxial tension are less brittle than the testing results. An explanation for this point is that: because of the localization of the damage zone into a macro-crack at the post-peak loading stage, the testing result is actually a kind of structural response which is a combination of the damage-caused snap-through softening in the plastic-damage zone, and the elastic unloading (i.e. snap-back) response outside the plastic damage zone corresponding to the displacement/strain loading at the end of the specimen. Consequently, for the constitutive behaviour of plastic damage investigated here, the local behaviour should be less brittle then the experiment results. Otherwise, a predicted structural response could be too brittle. Thus, this *less-brittle* property is also adopted by Lee and Fenves (1998) for tensile constitutive behaviour. The numerical results of damage-strain response shown in figure 2 grow asymptotically with strain loading.

3.3.2. Uniaxial Compression

The following Figures 3 and 4 show the stress-stain behaviour and damage evolution response of the proposed model under uniaxial compression.

It is seen in figure3 that the numerically predicted response of $\varepsilon_{11} - \sigma_{11}$ is in reasonably good accordance with the test results introduced in Lee and Fenves (1998) at both the pre-peak and post-peak loading stages. The $\varepsilon_{22} - \sigma_{11}$ curve shows the dilatancy property of the proposed model. In Figure 4, the damage evolution behaviour obtained numerically tends asymptotically to its limit value of 1.0.

3.3.3. Compression with Confinement

The behaviour of a model for concrete under hydro-static confinement is an important consideration as it indicates the pressure-sensitivity behaviour of the model. In the numerical tests performed in this study, the hydro-static confinement, i.e. $\sigma_m \mathbf{I}$, is applied before strain loading is applied in the 11-direction. Figure 5 shows the variation of the stress-strain response caused

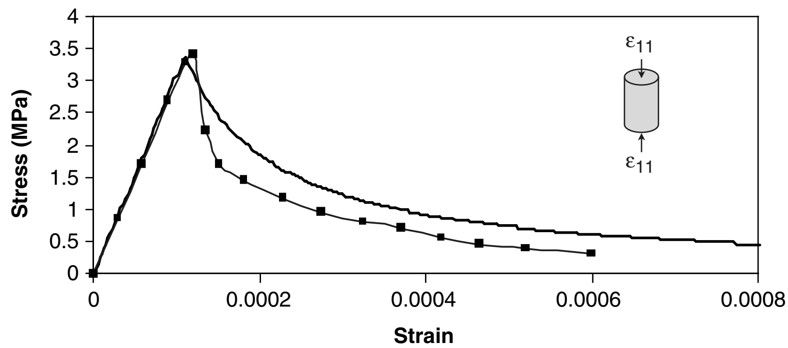

Figure 1. Stress-strain behaviour under uniaxial tension: the solid line represents numerical results and the black square marks represent experimental values (after Lee and Fenves, 1998)

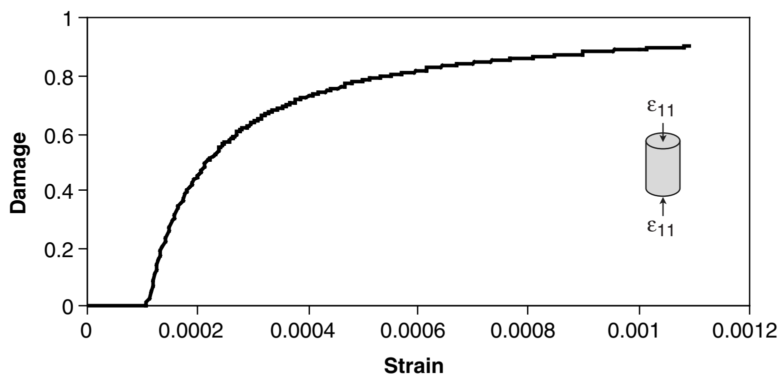

Figure 2. Damage evolution under uniaxial tension: numerical results by the model presented

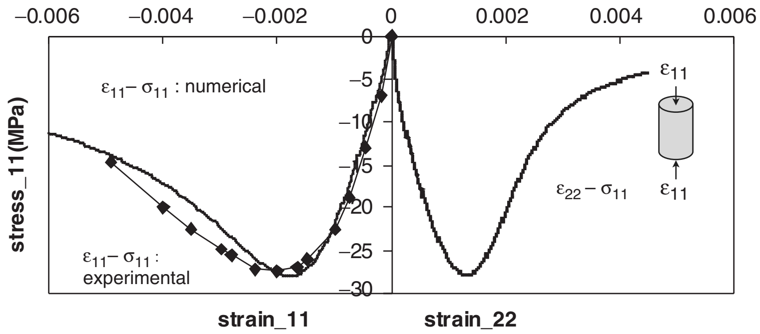

Figure 3. Stress-strain behaviour under uniaxial compression: the black squares represent experimental values (after Lee and Fenves, 1998), other curves represent numerical results

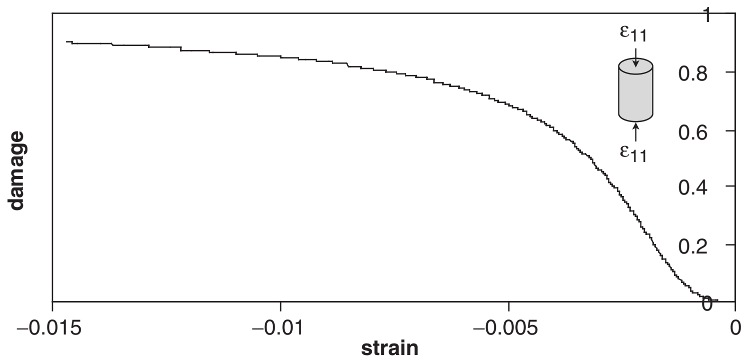

Figure 4. Damage evolution under uniaxial compression: numerical results

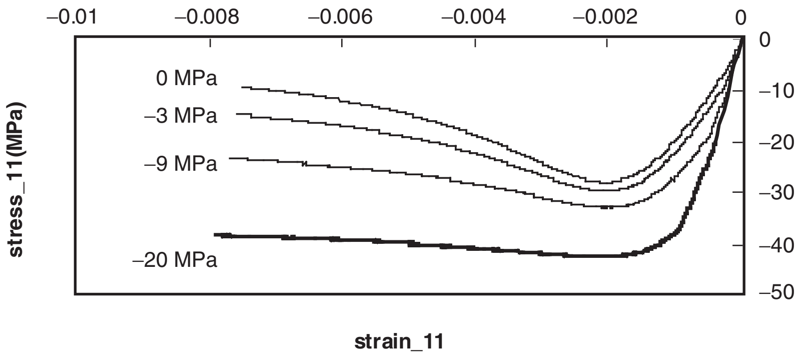

Figure 5. Influence of confinement: stress-strain behaviour (σ_m = 0, –3, –9, –20 MPa)

by the confinement of the stress-stain behaviour and damage evolution response, with the other parameters kept unchanged. As the increment of confinement increases, the softening phenomena become weaker and weaker.

4. CONCLUSIONS

Numerical validations have been made of the proposed plasticity-based damage model using the driver subroutine developed in this study. The behaviour predicted by the proposed model seems reasonably accurate for the uniaxial tension and uniaxial compression cases when compared with existing experimental data. Because of the high nonlinearity existing among the model parameters and the experimental phenomena, it is necessary to choose values for the constitutive parameters with some kind of inverse analysis technique.

It should be noted that it is easier to relate the stress-strain curve of the uni-axial tension test with the concept of Fracture Energy (for this case it is mode-I fracture energy), and also for the uni-axial compression test (for this case it should be the mode-II fracture energy because the Drucker-Prager criterion is basically a shear-type failure criterion). It is also reasonable to relate the compression under confinement curve shown in Figure 5, which has a softening branch, to Fracture Energy. However, it is impossible to directly relate to Fracture Energy those multi-axial phenomena which have no obvious softening branch. A reasonable explanation for this situation is that many cracks exist during multi-axial compression (i.e., compression with confinement), and Fracture Energy in a sense of a material constant does not work in that case.

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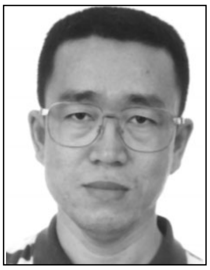

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