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# Investigation on Mixed-Mode Cohesive Crack in the Presence of Water Pressure for Interface Between Dam and Its Foundation

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**Abstract:** Firstly the focus is put on the formulation of an holonomic (in the sense of plastic deformation theory) constitutive model for a cohesive interface crack at the interface of a concrete structure accounting for the influence of water pressure. Secondly, the consequent discretized formulation of piece-wise linear interface laws in terms of generalized variables and its formulation as a linear complementarity problem is presented. At the end of this paper, numerical analysis for mixed-mode cohesive fracture of a concrete-rock interface of a benchmark gravity dam is described.

**Key words:** mixed-mode fracture, cohesive crack, interface, dam engineering, water pressure

## 1. INTRODUCTION

With the increase of needs for electricity and irrigation, more and more high dams have been built and there are at present several thousand high dams around the world. Consequently the safety assessment and failure analysis of dams has become more and more important a topic in recent years.

Interface joints between concrete dam and its rock foundation, as shown in Figure 1 is one of the potential sites of crack growth which eventually lead to the formation of conduits for water to seep through and exert uplift pressure. Besides this most important kind of joint, dams also consist of various other kinds of discontinuities. Some of them are unintentional zones of weakness such as horizontal construction joints; others are designed so as to temporarily or permanently accom-modate thermal strain, differential settlement and structural movement favorable to the intended load bearing capacity. In order to prevent concrete from possible cracking, various kinds of water-sealed heel joints have been designed on arch dams, of which the most

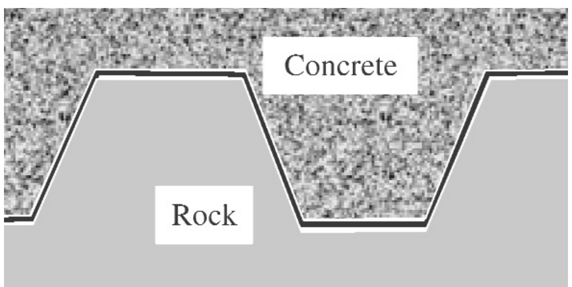



**Figure 1.** Illustration of artificial joints between concrete dam and rock foundation

prominent is complete separation of the arch dam from a peripheral foundation strip known as a concrete saddle or pulvino.

Generally speaking, the interface between two different materials and/or between two different blocks connected through cemented materials is considered to be one of the most important regions governing the strength and stability of certain structures. Different researchers in their analysis of concrete dams have used various interface models in recent years. As reported by references, existing constitutive models suitable for numerical analysis on joints of dams can be roughly classified into 3 groups: (1) Mode I cohesive crack models (see Guinea, Planas and Elices 1994; Bolzon, Maier and Novati 1994); (2) Mixed mode cohesive crack models, i.e. Coulomb-type elasto-plastic models (see Carol, Prat and Lopez 1997; Lofti, Shing 1994), and (3) elasto-plastic constitutive models for joint elements (see Goodman, Taylor and Brekke 1968; Hohnborg 1992) which *a priori* assume the existence of a discontinuity locus.

With the introduction of cohesive crack models assuming the existence of a fracture process zone ahead of macro crack for concrete fracture, many researches have been devoted to analysis of the fracture of cement-based materials, subjected to either pure tension or mixed-mode fracture loading. Cohesive interface crack models for simulation of fracture process in frictional concrete materials consist of the following two fundamental features: (1) Linear -elastic material behaviour is assumed throughout the considered solid or structure except at the locus of potential discontinuity. (2) Displacement discontinuity is allowed over that locus and is related to traction across it by a suitable relationship. A review (updated to 1991) of abundant literatures concerning cohesive crack model can be found in Bazant and Cedolin (1991) and more can be found in Karihaloo (1995).

Cracking and debonding of interfaces jeopardizes the integrity of a dam and its foundation. Water can then influence the opening and sliding of a discontinuity. Hydraulic pressure exerting at the concrete–rock interface is an important unfavourable factor to the safety of gravity dams, and the distribution of hydraulic pressure in the process zone is hard to be determined experimentally, but the popular opinion in engineering is that the value of hydraulic pressure in the process zone is dependent on the crack mouth opening displacement. Several researchers have investigated this topic in recent years (see Bruhwiler and Saouma 1995a and 1995b; Cocchetti 1998). In the work reported by Cocchetti (1998), a linear law is proposed for the distribution of water pressure in the fracture process zone. This linear assumption retains most of the principal characteristics of the influence of water pressure, and makes the related calculation highly simplified.

In the following sections, firstly, the definition of interfacial variables for a dam-foundation interface is given. Then the constitutive model for mixed-mode crack of concrete-like materials reported in Shen (2001) is briefly restated and extended to the case in the presence of water-pressure in the fracture process zone. An assumption of linear distribution of water-pressure in the process zone proposed by Cocchetti (1998) is adopted in this model. Based on the general principles described in references (see, e.g., Comi, Maier and Perego 1992), in Section 5, the discretized version in terms of Generalized Variables for mixed-mode interface fracture as a linear complementarity problem is presented. In Section 6, a numerical example is presented for the purpose of checking the capability of the proposed model in simulating the mixed-mode interface fracture in dam engineering.

## 2. DEFINITIONS OF VARIABLES FOR A DAM-FOUNDATION INTERFACE

At the interface between dam and rock-foundation the following vectors and relationships are defined (see Figure 2):

$$\mathbf{p} = \mathbf{p}^- = -\mathbf{p}^+ \qquad \mathbf{w} = \mathbf{u}^+ - \mathbf{u}^- \tag{1}$$

$$\mathbf{p} = \begin{Bmatrix} p_n \\ p_t \end{Bmatrix}, \qquad \mathbf{w} = \begin{Bmatrix} w_n \\ w_t \end{Bmatrix} \tag{2}$$

where $\mathbf{p}^-$ is the traction vector calculated from the lower part of the solid and $\mathbf{p}^+$ is the traction vector calculated from the upper part of the solid by FEM, and $\mathbf{p}$ is the traction vector acting on the interface; $\mathbf{w}$ is the vector of displacement discontinuity across the interface crack; subscripts "n" and "t" denote the components in the normal direction and tangential direction respectively. The sign for the traction calculated with FEM is so defined that positive is for the traction components in the positive axial direction and negative for those opposite to its related axis. For interface traction components, positive is for tensile and negative for compression. According to the sign-convention adopted in FEM formulated with Generalised Variables and that for interface variables,

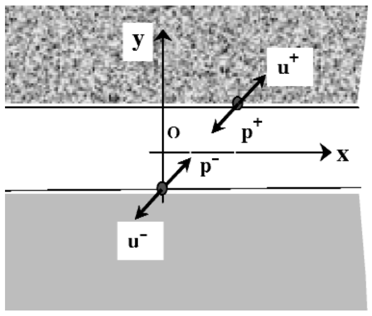



**Figure 2.** Illustration of interface variables

with Figure 2 we have the fundamental relationship expressed in Eqn 1.

## 3. PLASTIC YIELDING SURFACES AND INFLUENCE OF HYDRAULIC PRESSURE

The yielding surface for concrete-like materials presented by Shen (2001) is adopted. It is a piece-wise linear yielding model of 5 pieces of linear yielding surfaces, as shown in Figure 3, and its matrix form for a given traction vector is

$$\varphi = \mathbf{N}^T\mathbf{p} - \mathbf{H}\lambda - \mathbf{Y} \leq \mathbf{0} \tag{3}$$

The expressions for the matrices and vectors in Eqn. (3) are briefly stated in the following context. The vector of yielding functions adopted in this model is

$$\varphi = \{\varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \varphi_4 \quad \varphi_5\} \tag{4}$$

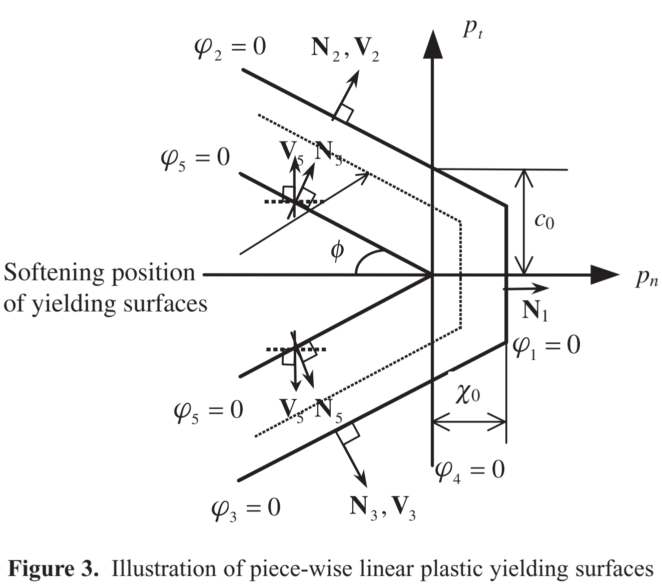



**Figure 3.** Illustration of piece-wise linear plastic yielding surfaces

The matrix of collection of normal directions of the first 3 yielding surfaces is

$$\mathbf{N} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_2 & \mathbf{N}_3 & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 1 & \mu & \mu & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix} \tag{5}$$

where $\mu$ is the frictional coefficient of interface material. The matrix of displacement softening modulus is

$$\mathbf{H} = \begin{bmatrix} H_n & \frac{\chi_0}{c_0}H_t & \frac{\chi_0}{c_0}H_t & -H_n & \frac{\chi_0}{c_0}H_t \\ \alpha H_n & H_t & H_t & -\alpha H_n & -H_t \\ \alpha H_n & H_t & H_t & -\alpha H_n & -H_t \\ H_n & \frac{\chi_0}{c_0}H_t & \frac{\chi_0}{c_0}H_t & -H_n & \frac{\chi_0}{c_0}H_t \\ \alpha H_n & H_t & H_t & -\alpha H_n & -H_t \end{bmatrix} \tag{6}$$

where $H_n$ is the softening modulus in the normal direction and it depends on the mode I fracture energy; $H_t$ is the softening modulus in the tangential direction and it depends on the mode II fracture energy; $\alpha$ is the influence coefficient of the softening of tensile strength on the softening of shearing strength. The vector of plastic multipliers is

$$\lambda = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5] \tag{7}$$

The vector of initial strength parameters of both tensile and shearing behaviour is

$$\mathbf{Y} = [\chi_0 \quad c_0 \quad c_0 \quad \chi_0 \quad c_0] \tag{8}$$

where $\chi_0$ and $c_0$ are the initial tensile and shearing strength respectively.

Since the hydraulic pressure in the process zone being accounted for, the plastic yielding conditions must be expressed in the space of total traction. In this calculation, Terzaghi's assumption is adopted and there is

$$p_n = p'_n - p_n^{(f)} \qquad p_t = p'_t \tag{9}$$

where $p'_n$ is the normal component of effective traction of the skeleton and $p_n$ is the normal component of total traction. Water pressure in the process zone, i.e. $p_n^{(f)}$, does not influence the distribution of traction component

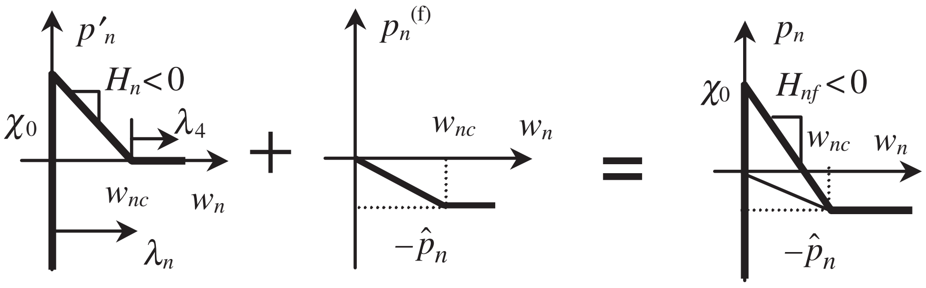



**Figure 4.** Illustration of influence of $p_n^{(f)}$ on the interface behaviour in normal direction

in tangential direction. According to the linear-assumption of water pressure inside the cohesive zone presented by Cocchetti (1998), as shown in Figure 4, we have for the mixed-mode crack the following expression for $p_n^{(f)}$:

$$p_n^{(f)} = \frac{w_n - \lambda_4}{w_{\mathrm{nc}}} \hat{p}_n \tag{10}$$

where $\hat{p}_n$ is the value of the hydraulic pressure at the mouth of the process zone, i.e. the tip of the macro-crack; $w_{nc}$ is the critical value of normal crack opening (also known as COD in fracture mechanics) and $w_n$ is the normal displacement at a point in the interface process zone; $\lambda_4$ is the 4th plastic multiplier in Eqn. (7) and its function here is to keep the value of $p_n^{(f)}$ at a point in the process zone to be not larger than $\hat{p}_n$.

After substituting Eqns 9–10 into Eqns 3–8, the matrix expression of yielding surfaces in terms of total traction at interface is obtained as follows:

$$\varphi = \mathbf{N}^T\mathbf{p} - \mathbf{H}_f\lambda - \mathbf{Y} \le \mathbf{0} \tag{11}$$

where

$$\mathbf{H}_f = \begin{bmatrix} H_{nf} & \left(\frac{\chi_0}{c_0}H_t - \frac{\mu\hat{p}_n}{w_{nc}}\right) & \left(\frac{\chi_0}{c_0}H_t - \frac{\mu\hat{p}_n}{w_{nc}}\right) & -H_{nf} & -\left(\frac{\chi_0}{c_0}H_t - \frac{\mu\hat{p}_n}{w_{nc}}\right) \\ \left(\alpha_n\frac{c_0}{\chi_0}H_n - \frac{\mu p_n}{w_{nc}}\right) & \left(H_t - \frac{\mu^2\hat{p}_n}{w_{nc}}\right) & \left(H_t - \frac{\mu^2\hat{p}_n}{w_{nc}}\right) & -\left(\alpha_n\frac{c_0}{\chi_0}H_n - \frac{\mu p_n}{w_{nc}}\right) & -\left(H_t - \frac{\mu^2\hat{p}_n}{w_{nc}}\right) \\ \left(\alpha_n\frac{c_0}{\chi_0}H_n - \frac{\mu p_n}{w_{nc}}\right) & \left(H_t - \frac{\mu^2\hat{p}_n}{w_{nc}}\right) & \left(H_t - \frac{\mu^2\hat{p}_n}{w_{nc}}\right) & -\left(\alpha_n\frac{c_0}{\chi_0}H_n - \frac{\mu p_n}{w_{nc}}\right) & -\left(H_t - \frac{\mu^2\hat{p}_n}{w_{nc}}\right) \\ H_n & \frac{\chi_0}{c_0}H_t & \frac{\chi_0}{c_0}H_t & -H_n & -\frac{\chi_0}{c_0}H_t \\ \alpha_n\frac{c_0}{\chi_0}H_n & H_t & H_t & -\alpha_n\frac{c_0}{\chi_0}H_n & -H_t \end{bmatrix} \tag{12}$$

$$H_{nf} = H_n - \frac{\mu\hat{p}_n}{w_{nc}} \tag{13}$$

In Eqn 12, the off-diagonal items of **H** are for the coupled influence of the softening in the normal direction to properties in the tangential direction of the interface crack and vice-versa.

## 4. CONSTITUTIVE LAW

The holonomic elasto-plastic constitutive law of the interface variables is expressed with effective traction as follows:

$$\mathbf{p}' = \mathbf{K}(\mathbf{w} - \mathbf{V}\lambda) \tag{14}$$

where **K** is the elastic stiffness matrix of the interfacial material and

$$\mathbf{K} = \begin{bmatrix} k_n & 0 \\ 0 & k_t \end{bmatrix} \tag{15}$$

where $k_n$ and $k_t$ are expressed as

$$k_n = E/t, \qquad k_t = G/t \tag{16}$$

where $E$ is the Young's modulus and $G$ is the shear modulus, and $t$ is the thickness of the interface layer. If $t = 0$, then $k_n = k_t = \infty$. The matrix $\mathbf{V}$ in Eqn. (14) is the collection of plastic flow vectors of all the 5 yielding surfaces, and is expressed by Shen (2001) as

$$\mathbf{V} = \begin{bmatrix} 1 & \mu & \mu & 0 & -\mu \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix} \tag{17}$$

By adopting the non-associated flow rule Eqn 14, together with Eqn 17, form the interface model of limited dilatancy.

## 5. SPACE DISCRETIZED FORMULATION OF INTERFACE MODEL

General principles for the space-discretized version of governing equations in terms of generalized variables for elasto-plastic structural problems can be found in Cocchetti (1998) and Bolzon, Maier and Tin-Loi (1997) and Comi, Maier and Perego (1992). In the following context, the space-discretized version of the governing equations of cohesive fracture problems in terms of generalized variables at an interface is presented.

Following the features of the interface models, it is assumed that the concrete dam behaves linear-elastically throughout the structure except at the concrete–rock interface. For the discretized nodal variables on the concrete–rock interface, there is

$$\bar{\mathbf{p}} = \bar{\mathbf{k}}\bar{\mathbf{w}} + \bar{\mathbf{p}}_g + \bar{\mathbf{p}}_h + \beta\bar{\mathbf{p}}_o \tag{18}$$

where $\bar{\mathbf{k}}$ is the condensed stiffness of material points at interface, and $\bar{\mathbf{p}}_g$, $\bar{\mathbf{p}}_h$, $\beta\bar{\mathbf{p}}_o$ are nodal force vectors related to gravity, hydraulic pressure, overtopping load respectively and can be obtained by finite element calculation; β is the loading factor of a reference overtopping load $\bar{\mathbf{p}}_o$. The interface traction is so defined that positive represents tension and negative for compression, and for tractions calculated from FEM, positive is for the one in the positive direction of the global coordinates. It is obvious that, if there is no crack opening, i.e., $\bar{\mathbf{w}} = \mathbf{0}$, then there would be the following elastic relationship at the interface:

$$\bar{\mathbf{p}} = \bar{\mathbf{p}}_g + \bar{\mathbf{p}}_h + \beta\bar{\mathbf{p}}_o \tag{19}$$

Accounting for Eqn (1) and the sign of $\bar{\mathbf{k}}$, we have

$$\bar{\varphi} = \bar{\mathbf{N}}^T\bar{\mathbf{p}} - \bar{\mathbf{H}}\bar{\lambda} - \bar{\mathbf{Y}} \tag{20}$$

Suppose that there are $n$ nodes at the interface, the matrices and vectors appearing in Eqn (20) can be specialized as follows:

The vector of collection of yielding functions is

$$\bar{\varphi} = \{\varphi_1^{(1)} \quad \varphi_2^{(1)} \quad \ldots \quad \varphi_5^{(1)} \quad \ldots \quad \varphi_5^{(n)}\} \tag{21}$$

$$\bar{\mathbf{N}}^T = \int_{\Gamma d} \bar{\mathbf{\Psi}}_\lambda^T \mathbf{N}^T \bar{\mathbf{\Psi}}_t \mathrm{d}\Gamma = \mathrm{diag}[\mathbf{N}^T, \ldots] \tag{22}$$

$$\bar{\mathbf{H}} = \int_{\Gamma d} \overline{\mathbf{\Psi}\lambda}^T \mathbf{H}^T \bar{\mathbf{\Psi}}_\lambda \mathrm{d}\Gamma = \mathrm{diag}[\mathbf{H}^T]\left(\int_{\Gamma d} \bar{\mathbf{\Psi}}_\lambda^T \bar{\mathbf{\Psi}}_\lambda \mathrm{d}\Gamma\right) \tag{23}$$

$$\bar{\mathbf{Y}} = \int_{\Gamma d} \bar{\mathbf{\Psi}}_\lambda^T \mathbf{Y} \mathrm{d}\Gamma \tag{24}$$

where $l$ is the distance between two interfacial points along the interface; $\bar{\mathbf{\Psi}}_\lambda$ is the shape (interpolation) function vector for displacement-relevant variables and $\bar{\mathbf{\Psi}}_t$ is the shape (interpolation) function vector for traction-relevant variables. In this article, linear interpolation function is adopted for both displacements and tractions at the interface.

Combining Eqns 9, 14, 18 and 20, there is

$$\bar{\varphi} = -[\bar{\mathbf{N}}^T\bar{\mathbf{k}}\bar{\mathbf{V}} + \bar{H}]\bar{\lambda} - \bar{N}^T(\bar{\mathbf{p}}_g + \bar{\mathbf{p}}_h + \beta\bar{\mathbf{p}}_o) - \bar{\mathbf{Y}} \tag{25}$$

The holonomic expression for the propagation of interface crack as a linear complementarity problem for $\bar{\varphi}$ and $\bar{\lambda}$ can be written as follows:

$$\bar{\varphi} = -[\bar{\mathbf{N}}^T\bar{\mathbf{k}}\bar{\mathbf{V}} + \bar{H}]\bar{\lambda} - \bar{N}^T(\bar{\mathbf{p}}_g + \bar{\mathbf{p}}_h + \beta\bar{\mathbf{p}}_o) - \bar{\mathbf{Y}} \tag{26}$$

$$\bar{\varphi} \leq \mathbf{0}, \quad \bar{\boldsymbol{\lambda}} \geq \mathbf{0}, \quad \bar{\varphi}^T\bar{\boldsymbol{\lambda}} = \mathbf{0} \tag{27}$$

The problem of interface fracture under the given traction vectors $(\bar{\mathbf{p}}_g + \bar{\mathbf{p}}_h + \beta\bar{\mathbf{p}}_o)$ can then be solved by the procedure of mathematical programming entitled as 'PATH Solver' by Dirks and Ferris (1995).

## 6. NUMERICAL APPLICATION

For the purpose of illustrating the capacity of the interface model described above, numerical analysis for the fracture of the interface between a benchmark dam and its foundation has been carried out. The geometry of the Benchmark gravity dam is shown in Figure 5.

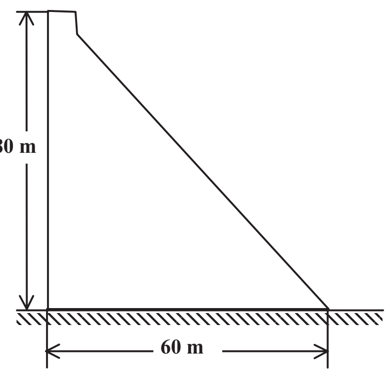



**Figure 5.** The geometry of the benchmark gravity dam


**Figure 6.** Illustration of interface nodes

Discretization of the 60-meter long concrete–rock interface has been carried out with a finite element program. Matrices $\overline{\mathbf{k}}, \overline{\mathbf{p}}_g, \overline{\mathbf{p}}_h, \overline{\mathbf{p}}_o$ were then generated by the finite element program for a calculation with a reference overtopping pressure $\overline{\mathbf{p}}_o = 1\,\text{MPa}$.The elastic modulus, Poisson's ratio and the mass per cubic meter for concrete are $E = 24000\,\text{MPa}$, $\nu = 0.15$, $\gamma = 24000\,\text{kg/m}^3$ respectively. A rigid rock foundation is assumed in the generation of the matrix $\overline{\mathbf{k}}$. A total of 61 interfacial nodes are uniformly distributed along the interface, and it is clear that the distance from a node to the crack mouth is $X$=(nodal number−1) meters.

## 6.1. Material Parameters and Model Parameters at Interface

Rigid-plastic interface material is assumed at first. Values of the other parameters are:

Frictional angle $\phi = \arctan(0.7)$, $\mu = 0.7$; tensile strength and shear strength are $\chi_0 = 0.3\,\text{MPa}$, $c_0 = 1\,\text{MPa}$ respectively; softening modulus in the normal and tangential directions are $H_n = -0.5\,\text{N/mm}^3$ and $H_t = -1.0\,\text{N/mm}^3$, which correspond with fracture energy for mode I crack $G_f^I = 0.09\,\text{N/mm}$ and for mode II fracture energy $G_f^{II} = 0.5\,\text{N/mm}$. The value of the coupling parameter is $\alpha = 0.8$.

The values of material parameters were so chosen that the initiation of the interface crack will start at a loading level lower than the full hydraulic pressure, i.e. the 80-meter level. This choice is made on the basis of information that this kind of case has been reported in dam engineering field (see Hohnberg, 1992).

## 6.2. Diagrams of Results with Hydraulic-Loading-Control

Calculations on the concrete–rock interface fracture of the benchmark dam have been carried out with aforementioned PATH solver and the holonomic 5-plane interface model expressed in Eqns 11–12. The resultant figures for the full hydraulic pressure together with a 5.15-meter overtopping water-head are shown in Figures 7 and 8.

Figure 7 indicates that the size of the process zone, in the sense of the area undergoing mixed-mode cracking (mathematically expressed as $\lambda_5 = 0$, $\text{Max}\{\lambda_1, \lambda_2, \lambda_3\} > 0$), is about 19 m long.

Figure 8 shows that with the given parameters and constitutive model, mode II cracking dominates the propagation of the concrete–rock interface crack. Experimental data is needed to verify this important phenomena, because if it is proved to be true for the general case, then conclusions based on mode I fracture analysis, which is popular in dam engineering, will become trivial, or at least become doubtful as a reference for the design of concrete dams.

Figure 9(a) and (b) show that the holonomic solutions of crack mouth opening in the normal and tangential directions respectively correspond to a series of values of static hydraulic pressure (between 48-m and 80-m water-head) and overtopping water-head (between 0.0 to 5.15 m).

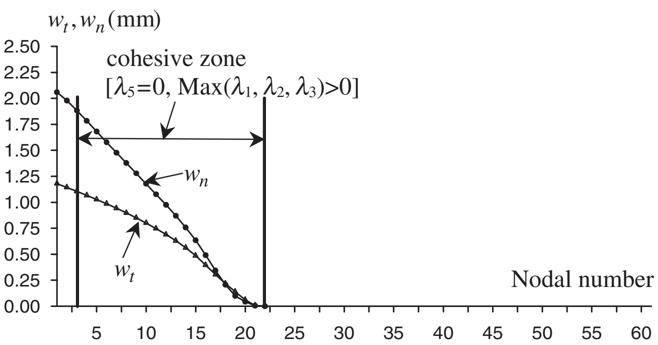



**Figure 7.** Distribution of crack opening displacements along the length of the interface

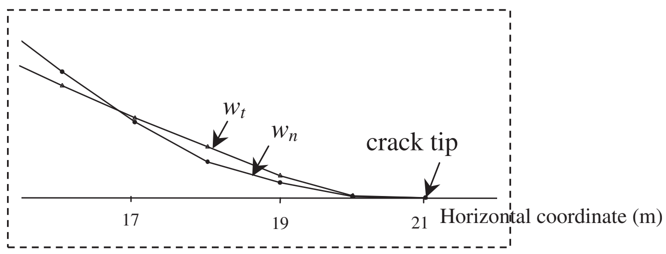
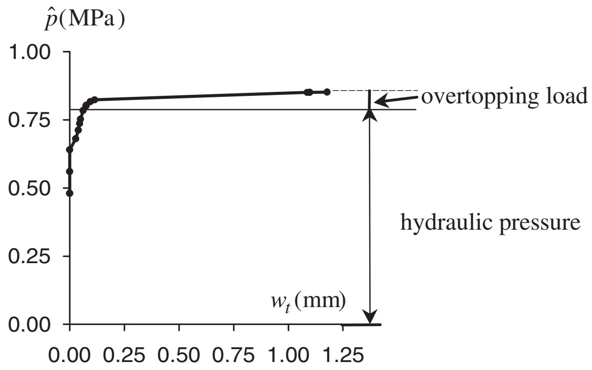
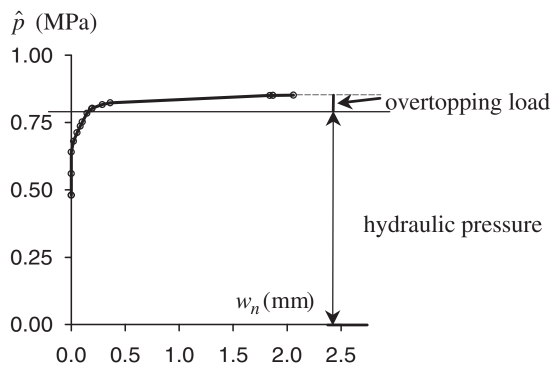



**Figure 8.** Distribution of displacements in the cohesive zone (enlarged local view)




(a) Response of load *vs* crack mouth opening in tangential direction




(b) Response of load *vs* crack mouth opening in normal direction

**Figure 9.** Diagram of load *vs* displacement (at the position of the first node, as shown in Figure 6)

It is seen from Figure 9 that, after the initiation of a cohesive crack at the concrete–rock interface in the presence of water pressure, there is a stable stage of crack propagation with increase of the overtopping pressure.

## 7. CONCLUSIONS AND ENDING REMARKS

With the piece-wise linear cohesive interface model, the discretized version in terms of Generalized Variables for mixed-mode interface fracture as a linear complementarity problem has been presented. Numerical analysis has been carried out for the mixed-mode quasi-brittle fracture of the concrete–rock interface between a benchmark dam and its foundation. The evolution of the interfacial cohesive zone has indicated that shearing fracture is the dominant factor for propagation of cohesive zone under the conditions adopted in this calculation. Solutions for a higher overtopping level have not been found yet using the subroutines adopted in this calculation. The difficulty of finding further solutions up to the stage when the interface crack has propagated to the whole length of the joint may be caused by the local multiplicity of solutions for internal variables (i.e. plastic multipliers) at the critical point at which instability such as snap-back would occur. Detailed discussion on the multiplicity and existence of solutions at the critical point with a more elaborate 7-plane piece-wise linear plastic yielding model can be found in Cocchetti, Maier and Shen (2001).

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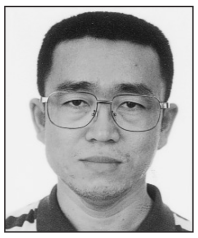

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