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A NEW NUMERICAL SCHEME FOR ELASTIC-PLASTIC SIMULATION OF EXCAVATION IN GEO-ENGINEERING

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Abstract For the path dependency and nonlinearity introduced by incremental construction, numerical method has been widely used in deformation analysis of geo-engineering. In the numerical simulation scheme commonly used in the past, the excavating loads are extracted from nodal stresses, which are deduced linearly from the stresses at Gauss-point in finite element method. The unneglectable calculation error is contained in this process when elastic-plastic constitutive model is employed. The error mentioned above is analyzed in detail. Based on the analysis of excavation process and the principle of finite element theory, a new simulation scheme for excavation is proposed. At the end of this paper, an application in rock engineering is given out.

Keywords elastic-plastic simulation, finite element method, geo-engineering, simulation scheme

Introduction

For a realistic evaluation of stresses and deformation in earth structures constructed sequentially, it is necessary to account for the path dependency and nonlinearity that introduced by incremental construction. The conventional approach based on linear material behavior of computing displacements and stresses as if the construction were completed in a single lift does not account for these factors nonlinearity in finite element formulation.

One of the approaches used commonly in geotechnical application was proposed by Desai[1]. We describe briefly the main features of approach in the following.

The in-situ stresses under gravity or overburden loading are first introduced into the discretized mass. The excavated surface is considered to be stress-free. Such a surface is assumed to be created by applying a set of equivalent forces at the nodes on the surface in the direction opposite to the direction of stressed due to initial and subsequent loading condition.

The increment stresses and displacements are evaluated by using the recursive formulas :

$$\{\sigma_i\} = \{\sigma_0\} - \sum_{j=1}^{i}\{\Delta\sigma_j\} , \quad \{\delta_i\} = \{\delta_0\} - \sum_{j=1}^{i}\{\Delta\delta_j\} . \tag{1}$$

A schematic representation of the calculation process of excavation is shown in Fig. 1.

In the algorithm described above , the excavating load vector q is extracted from nodal stresses. In displacement type finite element method , the results of the stressed values are the values at Gauss-point. It is necessary to deduce the value of nodal stresses vector σ_0 linearly from the values of stresses at Gauss-point. The unneglectable calculation error is contained in this process when elastic-plastic constitutive model being used. The error mentioned above will be analyzed in detail in the following section.

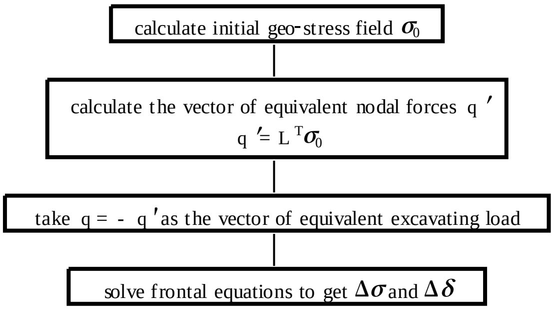

Fig. 1 Flow chart for numerical simulation of excavating processes which being used in the past

L is the outward normal vector

1 Analysis and basic principle

For an 8-node quadrilateral element as the second Gauss integration was employed , the formulas of calculating the stresses values at corner nodes is[2]

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{bmatrix} = \begin{bmatrix} a & b & c & b \\ b & a & b & c \\ c & b & a & b \\ b & c & b & a \end{bmatrix} \begin{bmatrix} \sigma^*_{\mathrm{I}} \\ \sigma^*_{\mathrm{II}} \\ \sigma^*_{\mathrm{III}} \\ \sigma^*_{\mathrm{IV}} \end{bmatrix} , \tag{2}$$

where , $a = 1 + \sqrt{3}/2$, $b = -1/2$, $c = 1 - \sqrt{3}/2$; $\sigma_1 \sim \sigma_4$ are the componental values of the vector of stresses at four corner nodes ; $\sigma_{\mathrm{I}} \sim \sigma_{\mathrm{IV}}$ are the componental values of the vector of stresses at Gauss-point.

As shown in Fig. 2 (a) , pure bending loads act a cantilever beam. If the deformation is limited in elastic status , the distribution of the stresses in the cross-section of the beam is linear (as shown in Fig. 2 (b)) .

Because the distribution of the stresses inside each elements are linear , the nodal stresses values calculated by equation (2) is accurate enough (Fig. 2 (c)) .

For the properties of low tensile strength of geo-materials , the elements on the excavated surface are always in elastic-plastic status under the action of excavation. For a beam , which is in elastic-plastic status , its stressed distributes broken -linearly in its cross-section (Fig. 2 (d)) .

The nodal stresses calculated by equation (2) are not accurate even if the elastic-perfectly-plastic constitutive model was used. The error status is shown in Fig. 2 (e) .

The softening constitutive model is usually used in elastic-plastic calculation for rock-like materials. As the elastic-brittle-plastic constitutive model being used , the distribution of the stressed in its cross-section is shown in Fig. 2 (f) . The status of errors is much more serious than that at elastic status (Fig. 2 (g)) .

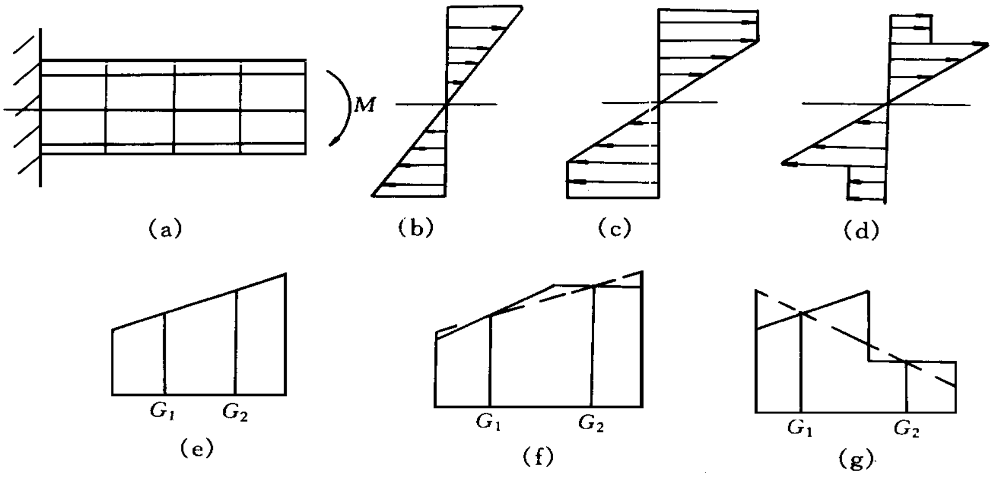

Fig. 2 Cantilever beam acted by pure bending load and the distribution of the stress in its cross-section

In view of above-mentioned facts, it is necessary to investigate the changing process of stress status in excavation process again to obtain an accurate solution. Fig. 3 shows a schematic representation of the changing process of the geo-stress field corresponding to excavation.

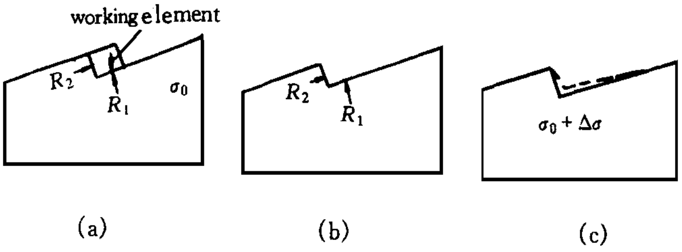

Fig. 3 The changing process of the equilibrium state in excavating process

In the primary state, the excavated elements serve the functions of the gravity action and prop action in order to equilibrate the original constructive stresses field and gravity stresses field. Being affected by all those forces mentioned above, the initial geo-stress field remains equilibrium. In the process of excavation, the equilibrium status of the geo-stress field remains equilibrium.

In the process of excavation, the equilibrium status of the geo-stress field is disturbed by the removement of the excavated elements. The new equilibrium status of geo-stress field after excavation is the combination of the disturbed original constructive stress field of the earth and the gravity field of the residual geo-structure.

In reality, the process of excavation is just the process of migration of equilibrium status under the action of unbalanced forces, The migration process of the equilibrium status in excavation process can be shown schematically in Fig. 3.

Fig. 3 (a) shows the situation that the actions (gravity and constructive action) of excavated elements on excavated surface is equilibrated by the reaction of the elements which jointed with excavated elements.

Fig. 3 (b) shows that: as excavated elements being removed, the elements, which jointed with them, lose their balance.

Fig. 3 (c) shows that the geo-structure arrive a new equilibrium status σ' and result in a relevant displacement field $\Delta\delta$.

In the process of elastic-plastic iterative calculation of finite element method which take the ratio error of the model residual unbalanced forces as its convergence criterion, the vector f of total unbalanced forces is regarded as the load vector for next iteration. These unbalanced forces include all the actions affecting the equilibrium of the structure , not only the gravity but also the action of construction. The elastic-plastic calculation equation of the unbalanced forces in finite element method is[3]

$$f = \sum \int_{\Omega} N^{T} G d\Omega - \sum \int_{\Omega} B^{T} \sigma^{(r)} d\Omega, \qquad (3)$$

where , f is the vector of unbalanced forces ; G is the vector of gravity loads ; $\sigma^{(r)}$ is the stresses field after r th iteration ; Ω is the region of the residual structure which not include excavated elements.

From the analysis described above , the following conclusion can be obtained.

The vector of unbalanced forces f obtained in the iterative calculation of elastic-plastic finite element method is just the vector of excavating load q acted on the excavated surface. Taking the ration error of the model of residual unbalanced forces as criterion , the calculation of the excavating load q , the unbalanced forces which caused by the elimination of the excavating elements , can be carried out by the calculation of plastic unbalanced forces f in the process of iterative solution of elastic-plastic finite element method. These unbalanced forces include not only the gravity of the excavated elements but also the plastic unbalanced force. For the case that the primary constructive prop stresses being not considered , the unbalanced forces caused by the elimination of excavated elements are mainly the action which being equilibrated by gravity.

Summing up all the analysis presented above , a rational numerical scheme for elastic-plastic simulation of the process of excavation can be obtained. This numerical scheme has been introduced in a finite element program , which mainly used for the stress-deformation analysis of geotechnical engineering. The main flow chart of that program is shown in Fig. 4.

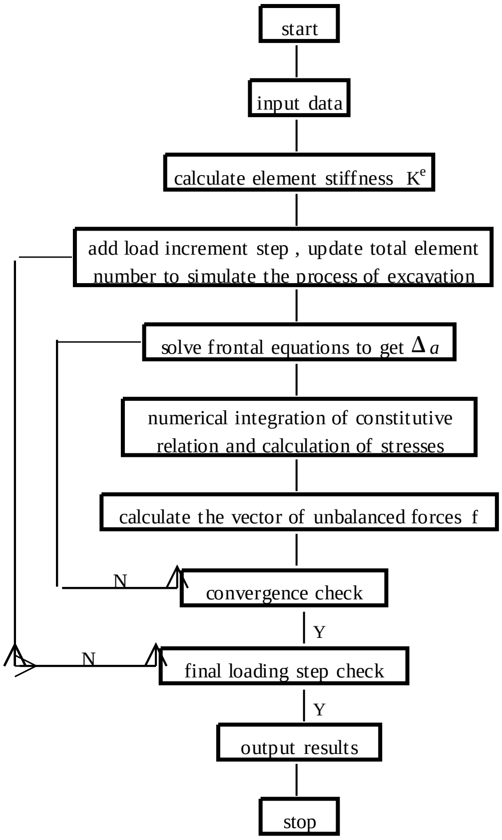

Fig. 4 Flow chart of the new scheme for simulation of excavation process

2 Application

The computer program developed in this paper has been used to analyze the displacement field of the sidewall of an open-pit mine. Take one section which normal to the sidewall as a two dimensional elastic-brittle-plastic structure of jointed rockmasses. Discrete the structure model by 6-noded elastic-plastic joint elements and 8 - noded quadrilateral elements. The corrected Mohr-Coulomb yield criterion , which has been smoothed at singular stress point , is used in the computation.

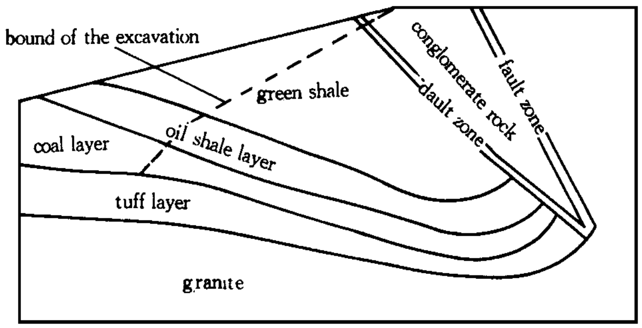

Fig. 5 Geo-mechanical model of the structure

The geo-mechanical model of rockmasses is shown in Fig. 5. The comparison between the numerical results and the measured data is shown in Fig. 6.

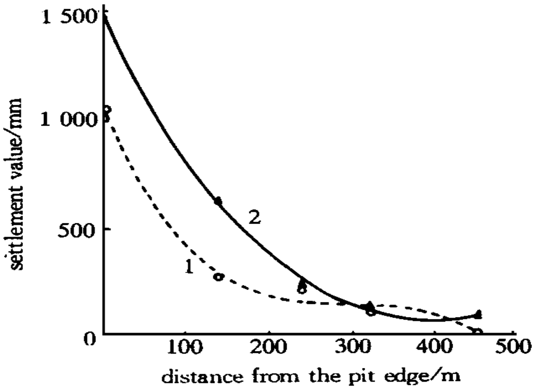

Fig. 6 Comparison between the numerical results and the measured data for the ground settlement during 1987～1990

1 ——measured data; 2 ——numerical results

3 Conclusion

Detailed analysis has been made on the mechanism of the process of excavation and the numerical scheme for the simulation of excavation. On the basis of the analysis described above and the basic numerical equations of finite elements in plasticity, a rational numerical scheme for the simulation of excavation has been proposed. Application in rock engineering is given out. The comparison between the numerical results and the measurements proves that the numerical scheme presented in this paper is practical and efficient.

References

1 Desai C S, Christain J T. Numerical method I geotechnical engineering. McGraw-Hill Inc, 1997. 52

2 Zienkiewicz O C. The finite element methods MC Graw-Hill, 1977

3 Owen D R J, Hinton E. Finite elements in platicity-theory and practice. Pinerige Press Limited, 1980. 180